VIBRATION ISOLATION OF FINITE-IMPEDANCE FOUNDATIONS BY PARALLEL RUBBER MOUNTINGS

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CERTIFICATE

This is to certify that the work entitled, "VIBRATION ISOLATION OF FINITE-IMPEDANCE FOUNDATIONS BY PARALLEL RUBBER MOUNTINGS" by B. Ravindra has been carried out under my supervision and has not been submitted elsewhere for a degree.

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-B. Ravindra

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LIST OF PRINCIPAL SYMBOLS

A	Coefficient of modulus of foundation mobility
$E^*_{\omega,\theta}$	Complex Young's modulus dependent on temperature and frequency
E_ω	Dynamic Young's modulus dependent on frequency
F	Force
$G^*_{\omega,\theta}$	Complex shear modulus dependent on temperature and frequency
G_ω	Dynamic shear modulus dependent on frequency
G_{P_ω}	Dynamic shear modulus of parallel mount
I_y	Moment of inertia
K^*	Complex stiffness of the mount
K	Geometric parameter of stiffness
M	Mass of machine
M_s	Mass of steel
M_R	Mass of rubber
P	Power flow
Q_f	Power flow coefficient
R	Response ratio
T	Transmissibility
V	Velocity
a	Cross-sectional area
a_1	Coefficient of dynamic modulus
b_1	Coefficient governing frequency dependence of dynamic shear modulus
ℓ	Length

n	Number of periodic elements
s	Phase of foundation mobility
t	Time
x,y,z	Translational displacements along coordinate axes x,y,z respectively
α',β',γ'	Rotations about coordinate axes x,y,z respectively
β	Foundation mobility
β_{ij}	Mobilities of periodic isolation system
γ	Normalised real part of foundation mobility
γ_1	Ratio of M_S to M_R
γ_2	Ratio of M to M_R
δG_{ω}	Damping factor corresponding to shear modulus
ΔP_{ω}	Damping factor of parallel mount
δ	Normalised imaginary part of foundation mobility
ρ	Density of the material
ρ_y	Radius of gyration
$\emptyset$	Phase difference
ω	Frequency of excitation
ω_o	Natural frequency.

ABSTRACT

The main objective of the present work is to investigate the effects of foundation mobility on vibration isolation of machines supported by parallel rubber mountings. Different types of finite impedance-foundations are analysed, as if they were infinite in extent and expressions for the response ratio and vibration power flow are derived to evaluate the effectiveness of vibration isolation. Frequency dependent properties of rubber-like materials are considered and their effect on vibration isolation is examined. Systems with some kind of periodicity, namely, periodic vibration isolation systems and periodic isolator are discussed. A mobility method is used to analyse the performance of these periodic systems on finite-impedance foundations. Wave effects in isolators on vibration isolation are analysed. Analysis of vibration isolation for two point excitation model is carried out.

CHAPTER I

INTRODUCTION

1.1 Introduction

Vibration control engineering is moving at a rapid pace associated with the progress in other technical areas. Machines running at high speeds, light weight machine and vehicle structures, performance sensitivity of electronic and optical equipment are the catalysts in the growth of vibration control engineering. The growing importance of ergonomic aspects in designing shop floors and buildings etc., to create a safe and pleasant working environment devoid of noise and vibration is another source of motivation for the progress in vibration control.

In many engineering applications, need for vibration control arises due to the unbalanced forces in machinery and due to the vibrating foundations.. The nearest that one can get to control the vibration is to control it at the source itself. But in many practical problems, it is very difficult to completely eliminate all the vibration arising at the source and it may even prove to be cost ineffective. Therefore,it is apparent that secondary methods of vibration control are necessary to tackle many vibration problems. Vibration isolation is one such important and widely used method.

The function of vibration isolation primarily is to reduce the magnitude of motion transmitted from a vibrating foundation to the equipment, or to reduce the magnitude of force transmitted from the equipment to the foundation. Depending on the method of producing the controlling force of vibration, vibration isolation can be classified as passive, active, semiactive, hybrid etc. In active vibration isolation the isolator force acting on the mass is servo-controlled. This results in a better overall reduction in transmission of vibration. But this is limited to light weight machines. If the isolator force is not controlled externally, then the method is referred to as passive vibration isolation. Combinations of the above two methods give rise to semiactive and hybrid vibration isolation methods.

A vibration isolator in its most elementary form may be considered as a resilient member connecting the equipment and foundation. Many different materials are used as the resilient materials of isolators. Isolators can be rubber springs, plastic springs, metal springs, air springs etc. Each one of these has some advantages and disadvantages. The damping of the isolator is inherent in some materials like rubber and provided externally in some cases like metal springs. Several types of damping mechanisms such as viscous, hysteritic, Coulomb, hydraulic, air, velocity to the nth power, are used. Elastomer isolators find wide applications because they may be conveniently molded into many desired shapes with tailor-made stiffnesses and internal

damping. These also usually require minimum of space and weight, and can be bonded to metallic inserts adapted for simplified attachment to the isolated structures. Some of the disadvantages are the temperature and environment dependent properties of rubber.

The simplest idealisation of the vibration isolation system is a single degree-of-freedom model in which a mass is constrained to move only in one direction. The isolator is assumed to be massless and the foundation is considered as ideally rigid. More complex vibration isolation systems can be visualised by relaxing these assumptions, such as when (i) the foundation is not ideally rigid, (ii) the isolator inertia cannot be ignored, (iii) several masses with intermediate elastic elements are involved in the model or (iv) the mass can move in any direction like heave and roll etc. The latest works are considering the machine as a structure (no longer a rigid mass) connected to the foundation structure by isolators at several points. With the advent of modal analysis, these more complex problems can be tackled in an effective manner. But with increasing complexity, the problem becomes more specific and it is difficult to obtain generalized conclusions.

Further complications of the vibration isolation system may arise out of the type of excitation such as random or impulse excitation and due to the nonlinearities of the system.

Though the research on vibration isolation is vast, very little attention is paid to provide guidelines to design or select the isolators on a rational basis. The general

practice of designing of the isolators is still on the assumption that all will be well if the natural frequency of the isolator is appreciably lower than the frequency of vibration that is to be isolated. An isolator designed on this basis for a vibrating machine may prove useless, if the response of the supporting foundation is ignored. The need for considering the nonrigidity of the foundation is apparent in many engineering situations where machines are mounted on flexible structures such as shipdecks, aircraft fuselages, car or train chasis and building floors. Therefore the main objective in this work is to show how the flexibility of the foundation can be taken into account while analysing various vibration isolation systems.

1.2 Review of the Previous Work

The amount of literature available on the general problem of vibration isolation is too vast. In this section, the literature which has direct bearing with the present work is discussed. An overview of the research work in different areas, connected with the present problem, is categorized under the following headings:

(i) Static and dynamic properties of isolator materials

(ii) Stress analysis of rubber mounts

(iii) Modelling of finite impedance foundations

(iv) Analysis of a single point excitation model

(v) Analysis of an unidirectional multidegrees of freedom model

model

(vi) Analysis of two or multipoint excitation model

(vii) Wave effects in isolators

(viii) Experimental methods to measure the transmissibility and vibrational power flow.

Due to the usage of various equivalent terms and indices in the literature on isolation, a compendium of these is put in Appendix A for ready reference.

1.2.1 Static and dynamic properties of isolator materials

A detailed account of static and dynamic properties of rubber like materials is given by Snowdon [1,2]. A sample of materials sandwitched between plane, parallel, rigid surfaces subjected to compression is studied and relations between stress and strain interms of Young's modulus, shear modulus and bulk modulus are given. Apparent modulus for an element of rubber like material is given in terms of shape factor. Snowdon [2] has also given plots showing the dependence of apparent modulus on the shape factor and rubber hardnesses. Expressions for shape factor for simple geometries are given. Measured values of E, G, and B for natural rubbers of increasing hardness are listed. Creep of rubber under static load is examined and the load deflection and stress-strain curves for statically compressed rubbers are included.

The concept of complex elastic modulus is explained in reference [1]. The dynamic properties of rubber like materials that experience sinusoidal vibration are readily accounted for by writing the elastic moduli as complex quantities. Frequency and temperature dependence of the mechanical properties of rubber like materials (specifically dynamic modulus and damping factor) and the behaviour of these materials at transition frequency and transition temperature are explained. Dynamic modulus and damping factor versus frequency plots are given for some typical low damping and high damping rubber like materials. Temperature dependence of dynamic modulus and damping factor of some typical rubber materials is discussed. The addition of carbon black and the resulting amplitude dependence of dynamic modulus and damping factor is examined in reference [2]. The static load experienced by a rubber sample can also influence its dynamic properties. Plot of dynamic stiffness of a natural rubber sample versus static stress is given. A review paper published in 1986 [3], updates the earlier review paper on the behaviour of rubber like materials under sinusoidal forces. Most significant advances concern the correlation of dynamic properties and molecular structure. This review covers general properties, testing methods, dynamic data and applications.

The effects of strain amplitude, static or dynamic modulus and damping factor are reviewed in reference [4] with special reference to the design of antivibration mountings. The effect of addition of different types of carbon black and reinforcing fillers on vibration control is presented in reference [5].

The temperature field inside a vibrating rubber solid cylinder is investigated in reference [6]. The rubber cylinder, a specimen of a vibration isolator is subjected to cyclic compressive force by means of an electrodynamic shaker. The temperature at 16 different locations inside the cylinder are measured and compared with values of the temperature found from the analytical investigation.

The experimental research carried out for determining the elastic characteristics of a new elastic material, VIBRAMOR produced out of rubber waste, under the action of cyclic loading is reported in [7]. The elastic and damping properties of this material are found to suit the properties for the antivibratory isolation of machines and their foundations.

The effect of strain history on the dynamic modulus of carbon black filled rubber is given in reference [8]. A new elastomer blend with high damping, low modulus and high fatigue life is studied as a potential material for making an engine mount [9].

Marsh mellow elastomeric spring - a constant natural frequency spring irrespective of loading is studied for its feasibility as a antivibration mount. This eliminates the problems associated with overheating, excessive compression set and temperature sensitivity etc. [10].

The effect of nonlinearity in the dynamic behaviour of rubber is examined in reference [11]. The validity of using properties, obtained from sinusoidal excitation used in testing, for a service environment involving complex patterns of vibrations is studied. Random vibration tests with a continuous spectrum of vibrations are conducted. For more complex systems of vibrations, it is found out that nonlinear rubbers will behave in a more linear fashion and will exhibit higher damping than indicated by dynamic properties measured in conventional sinusoidal tests. When service environment of many components is taken into account the dynamic behaviour of nonlinear rubber gives better performance than expected from conventional testing.

Experimental methods to measure dynamic modulus and damping factor of rubber mounts are explained in [12,13]. Considerations in designing of elastomeric products for control of vibration are reported in [14,15].

A method for presenting dynamic performance characteristics of a highly damped vibration isolators is reported in

[16]. This method utilises normalised stiffness versus load graphs obtained from laboratory test data and enables one to determine the dynamic stiffness and natural frequency as a function of isolator load and temperature. Because similar types of isolators show similar isolation characteristics with respect to load and temperature, they can be categorized into families based upon material formulation and isolator geometry. This provides a means for presenting a large amount of performance data in an abbreviated format.

Since the properties of rubber like materials used in making isolators depend on many parameters like frequency, temperature, static strain, vibration amplitude etc, a suggestion is given in reference [17] to conduct dimensional analysis and facilitate the presentation of results in a brief format so that they can be used in further analysis and design of the system.

1.2.2 Stress analysis of rubber mounts

The following different methods have been attempted for the stress analysis of rubber mounts:

(1) Approximate formulae based on Hooke's law behaviour of rubber (strength of materials approach)

(2) Analytical methods based on theory of elasticity (plane stress, plane strain problems)

(3) Analytical methods taking the nonlinearity of rubber into account.

(4) Numerical methods like FEM and experimental methods like optical methods and dimensional analysis.

Geometries of various rubber mounts used in practice are given in detail in reference [18]. Simple formulae which relate stress and strain for simple geometries of rubber mounts subjected to various loading patterns are given by Gobel [19]. These results can be found in many standard books on vibration control [20, 21]. These expressions are valid so long as the deformation is within the linear range. The deformations associated with rubber mounts are generally small and moreover the tolerances in manufacturing rubber mounts may offset any advantage gained by complicated analysis [4]. Due to the nonavailability of analytical methods, engineering design of rubber springs is currently pursued with the seat of the pants technology.

Most of the research on rubber mounts is done for a specimen which consists of a rubber block bonded to two steel plates at its ends. A brief review of the work done on such mounts is given in reference [22].

An approximate analysis of infinitesimal deformations of bonded elastic mounts is given in reference [23]. An approximate simple theoretical solution is developed for infinitesimal, plane and axisymmetric strain deformations for

blocks, of elastic material with Poisson's ratio between 0 and 0.5, bonded to rigid end plates. The explicit form of solution, developed for shape factor, S, between 0 and ∞, is easy to use and compares well with experimental results. Later this analysis is extended to finite (nonlinear) deformations of bonded elastic mounts [24].

Equivalent spring model representation of a vibration isolator with distributed stiffness undergoing rotation oscillation is given in reference [25]. Expressions for effective stiffness of the two spring model for bonded rubber mounts when isolated mass performing rotational as well as linear vibration in the direction of excitation are presented.

A review paper by Ogden [26] covers the theory of rubber elasticity. He covers incompressible theory, compressible theory, inhomogeneous deformations of rubber like materials. Another paper by Beatty [27] is a survey of some selected topics in finite elasticity. He covers hyperelasticity of rubber, elastomer and biological tissues with examples.

A review of application of finite element analysis in design of elastomeric mounts is presented in reference [22]. Reluctance in using FEM is examined. Comparative closed form solution versus FEA for rubber mounts are given. Non-linear stress distribution and static force deflection response for a rubber mount are calculated. It is claimed that FEA provides more detailed information for design than that can possibly be measured on actual component part or computed using classical closed form solutions. Computer costs are

justified by factors such as efficiency and better component reliability and performance.

Tseng et al [28] has developed a nonlinear, incompressible element using a perturbed Lagrangian formulation to simulate incompressible or nearly incompressible elastomers. A FE program which is capable of handling material and geometric nonlinearities is developed and the application of FEA to improve on existing design is illustrated by using a engine mount as an example. Numerical and experimental results are compared.

An interactive FE program has been developed to predict the mechanical impedance of a homogeneous rubber slab over a rigid base [29]. The effect of design variations on impedance can be readily studied and the system can be optimised for a selected range of frequencies and impedances.

Method of caustics, an experimental method based on optics has also been used to evaluate stresses in a rubber mount [30]. A general introduction to the design of rubber mounts using FEM and optical methods is given in reference [12].

An empirical model for the design of rubber shear bushings is given in reference [31]. An experimental investigation is conducted to quantify the nonlinear behaviour of rubber springs. Investigation is focussed on axial quasi-static load/deflection properties (not dynamic) and a suggestion is made that the dynamic response can be obtained by extrapolating

from static conditions. Dimensional analysis is carried out to evaluate empirical formulae from experimental results.

Problems associated with the design of rubber springs are as follows [31]:

(1) Nonavailability of analytical methods of design. Apart from simplest geometries, statistical and phenomenological theories of large elastic deformation do not offer equations which can be solved in closed form.

(2) FEA requires large amount of computer time as rubber is nonlinear. Thus FEA in the design process is not one of providing a means for initial conceptualisation but rather, if the cost justified, a means of refining an idea.

(3) Experimentally obtained data is a source for determining empirical formulation which is good enough only as a first approximation for a limited range of applications.

1.2.3 Modelling of finite impedance foundations

In the conventional analysis of vibration isolation problems the foundation is assumed to be perfectly rigid and infinite in extent. These assumptions though valid in many practical situations, are not always applicable. For

example, vibration sources are mounted on ship, aircraft and automobile structures. Before reviewing the work on modelling the foundation, different types of foundations should be examined. A broad classification can be as follows:

(1) Soils and floors (as in buildings)

(2) Beams, plates, plates stiffened with beams (as in ships and aircrafts)

(3) Consisting of many structural elements (as in automobiles).

Generally the foundation is modelled by point mobilities or point impedances. Measurement of mobility and damping of floors for different types of wooden and concrete floors in occupied buildings in three frequency bands is done by White et.al. [32]. A vertically applied excitation is used. In the case of an infinite plate, the mobility may be estimated from the material properties and plate thickness. But for practical floor constructions, the boundary conditions and inhomogenities complicate the situation and it is therefore very difficult to estimate the mobilities quantitatively.

The need for considering the floor flexibility is clearly brought out by Macinante [33]. Practical examples and data are given to substantiate the necessity of considering the flexibility of foundation. The floor is modelled as an equivalent mass-spring-damper system. Determination of floor

characteristics used in the above model is explained.

Snowdon [1] has considered the driving point impedance at the center of a simply supported beam to simulate the mechanical properties of a nonrigid foundation. Later this analysis is extended to Plates [2].

Machines mounted on cantilever beams and the effect of modification in the foundation structure is studied by Wang et al. [34]. Similar analysis is also considered for a plate like foundation [35]. Nakra et.al. [36] has considered the foundation as a composite beam. Impedance relationships for a foundation structure composed of idealised finite length beams with various boundary conditions are reported in reference [37].

Though in practice, machines are sometimes mounted on large flexible structures having so many modes of vibration, it is not convenient to consider each mode separately. Therefore a simple finite seating is modelled and such a study is reported in reference [38], so as to find out when it can be approximated as an infinite seating in making frequency averaged power predictions.

In references [39-41], a number of typical foundations such as beams, plates etc. have been analysed as if they were infinite. Beams and plates with force and torque excitation are studied and the resulting near and farfield powerflow mechanisms are examined. The driving point mobilities for these elements are expressed in terms of simple functions of

frequency which yield straightline variation on log-log plot of mobility and frequency. A comparison between finite and infinite structures is made.

Pinnington and White [38] presents a parametric study of the modelling of a finite structure (in this case a cantilever beam) as an infinitely long beam and shows that the average power transmission is same in both cases except at resonance. This is not valid for periodic structures.

Goyder [42] examined various curvefitting procedures for modelling a structure. A structure with a number of resonances is difficult to analyse theoretically, but may be investigated using the measured data. By exciting a structure at one point and measuring the frequency response at a number of positions it is possible to construct a mathematical model of the structure. Methods of mathematical modelling are reviewed By modelling two separate components of a structure from measured data, it is possible to obtain an estimate of the subsequent motion and powerflow through the two components when coupled.

Modelling of a foundation for multipoint excitation is examined by Peterson and Plunt [43]. On the elementary case, i.e. with one contact point between the source and the receiving structure and one direction of motion, there is a single force and a single velocity present and accordingly the input power to the receiving structure is given as the real part of the complex product of force and velocity. However, this is rarely

found in practice, where often the components of the coupled system are heavy and large and have to be mounted at several contact points. In such cases, the interaction between different points and in general also between different directions must be taken into account. The properties of these multipoint, coupled systems can generally be matrix formulated and accordingly there is a 6N x 6N mobility matrix for each structure to handle where N is the number of contact points. It is possible to neglect the influence of some elements in the mobility matrix but it is an intricate problem to sort these out at an early prediction stage. The possibilities of rearranging the general mobility matrix into some corresponding effective mobilities have been investigated by using two principally different formulations on the concept of effective mobility. The two concepts of effective mobility, namely effective point mobility in which the points are considered individually with the interaction between the points taken into account and effective overall mobility in which a space averaged point mobility is deduced, have been studied theoretically in reference [43] and verified experimentally in reference [44]. In addition, some useful approximations of these quantities are derived.

Pinnington [45] presents a model of the foundation structure which supports a motor on four isolators using point accelerance and transfer acclerances.

Analysis of foundation structure which consists of many structural elements is done by standard structural dynamics methods like dynamic stiffness method or component mode synthesis methods. Dynamic stiffness method is used by Sainsbury and Ewins [46,47] for the multidirectional isolation of machinery vibrations. Each component of the foundation structure is analysed either theoretically or experimentally for its mechanical impedances and individual components are then connected together using the dynamic stiffness coupling technique in order to predict the vibration characteristics of the complete structure.

General method of modelling the structure, using direct and cross compliances and thereby analysis of two structures connected by several viscoelastic elements, is given in reference [48]. As an example, the foundation structure of a compressor is modelled as a plate and measured values of compliances are used in the analysis of the system.

1.2.4 Analysis of single point excitation model

Single point excitation model is nothing but a machine (mass) moving in a single direction connected to the foundation structure by a vibration isolator.

The problem of vibration isolation of mass from a infinite impedance foundation by using a vibration isolator has been studied extensively [1,20,18,49]. Researchers have used different indices to express the effectiveness of a vibration isolator. Some of them are:

(1) Force/displacement transmissibility (absolute)

(2) Relative transmissibility

(3) Response ratio

(4) Isolation effectiveness

(5) Insertion loss

(6) Vibration power flow

(7) Vibration power flow ratio.

Log-log plots of the above mentioned quantities v. frequency ratio can be found extensively in literature for various (different) system parameters.

The influence of different types of damping on vibration isolation is presented in detail by Ruzicka and Derby [49].

Snowdon [1] has studied this problem when the isolator is made of rubber like materials for rigid as well as non-rigid foundations. The frequency dependence of dynamic modulus and damping factor of rubber like materials is taken into consideration. Its effect on transmissibility of the system for low as well as high damping rubbers is studied. The concept of parallel combination of low and high damping rubber for better vibration isolation characteristics is explained. Snowdon has used the term 'response ratio' to quantify the effect of vibration isolation for nonrigid foundations. The nonrigid foundations considered are simply-supported beams and plates. The effect of mass loading of the foundation on response ratio has also been studied. This analysis is later extended to

the case when the feet of the machine supported on the isolator are nonrigid [2].

Macinante [33] considers a two mass model to take care of flexibility of floors and the transmissibility expressions (force transmitted to the structure supporting the floor) are derived in terms of non-dimensional parameters of the machine, isolator and floor. The way in which transmissibility is influenced by two frequency ratios, viz., mounting frequency ratio and floor frequency ratio, is clearly shown as a surface in a three-dimensional plot. A complete parametric study is conducted and the results are plotted graphically.

Nomograms have been designed to make it possible to obtain quickly, with reference to the most widely used techniques of vibro-isolation, a complete picture of the admissible solutions, in terms of the elastic-inertial-damping characteristics of the machine foundation system [50].

In reference [51], criteria of effective isolation of machinery are derived and design of isolators complying with these criteria are described. The concept of vibration power flow is explained by Goyder and White [4]. Expressions are derived for vibration power flow to the foundation for single point excitation, taking the foundation mobility into consideration. The constant force as well as constant velocity sources are considered. Design considerations for isolators

and foundations to minimize the transmission of vibration power in the structure are examined.

Pinnington and White [38] have also investigated the parameters controlling power transmission from machine to the seating structure via spring like vibration isolators. The effects which the seating has on the power input is studied. General expressions for the power input have been obtained of which two forms involve the real component of the seating mobility and impedance, respectively.

An alternative description of the power transmission from source to receiver is presented in the case of one component of motion and one contact point [52]. A general source parameter called the source descriptor is introduced. This parameter is a function of the source only. It is proportional to power and involves both the internal vibration of source and its mobility at the contact point. From experimental measurement series, it can be emphasized that the velocity of the free source is a suitable and manageable quantity for describing the internal vibration of many structure borne sound sources. The active power transmitted from the source structure to the receiving one is found to be the product of the source descriptor and a dimensionless function called the coupling function. The function illustrates the dynamic properties of the source - receiver interfaces. The idealizations of constant force source and constant velocity source are examined and it is shown that they do not offer a reliable

estimation when the source and receiver mobilities are comparable. It is shown that the idealizations of constant force and constant velocity sources provide asymptotic estimations of the transmission when the source and receiver mobilities differ greatly. In case of similar dynamic characteristics, the constant force and constant velocity assumptions are not fully valid. The concept of source descriptor and coupling function provide a general way of estimating the transmission without any assumption about the interface.

The effectiveness of structure borne noise isolation systems on a light weight ship foundation is reported in reference [37]. Impedance relationships are developed for the machine, sub-base, foundation, ship hull and the intervening isolators. The results of a parametric study to assess the effectiveness of vibration isolators, for various system parameters is presented for a ship hull of aluminium construction.

Selection procedure for resilient mounts protecting fragile equipments, considering foundation and equipment structural admittances, is given in [53]. A simple criterion for mount adequacy and overlay for rapid selection of mount stiffness and damping are given. Mount restrictions to avoid bottoming under sinusoidal and random excitations are also studied and discussed.

The overall methodology used to design foundations are sometimes a compromise between various conflicting requirements of noise, shock, vibration, ship motion,thermal motion and machinery imposed loads. The iterative process of structural optimization for shock and vibration is linked to noise attenuation performance. The analysis of noise transmission through foundation is reported to be carried out using the direct dynamic stiffness method in reference [54].

Vibration isolators for machines installed on ships are subjected to both dynamic forces involved with the working of these machines and inertial forces resulting from the rolling of the ship. In order to illustrate this better, a solution of the vibration equation of a single mass system has been analysed, the system being excited simultaneously by a dynamic force and a motion of the foundation. It has been found that the larger the ratio of rotation speed to natural frequency of vibration, the larger is the relative displacement of a machine mounted on vibration isolators. Soft isolators can be used,provided the frequency ratio mentioned above is chosen such that the elastic deformation of the isolators is within the allowable limits [55].

Methods of optimum design of isolators are reviewed in [56]. A study of the influence of nonlinear spring stiffness-characteristics on the effectiveness of vibration isolators with linear damping, subjected to stationary, random white noise ground acceleration, is presented in

reference [57]. Results of the random vibration analysis with the cubic hard spring, the cubic soft spring and the tangent spring are presented in a manner useful for engineers and designers of vibration isolation systems.

Complex motion of the isolated mass at high excitations taking into account, nonlinear material behaviour on transmissibility of vibration isolating systems has been studied and bifurcations are observed [58]. Beatty considers the finite amplitude, periodic motion of a body supported on rubber shear springs [59,60,61].

The standard approach to vibration isolation analysis represents the source, the isolator and the receiver by their mobilities. The choice of appropriate descriptor for each element is not arbitrary, specifically when the isolation system is nonlinear or when the system has noise. At least one of the system components must be represented by its impedance [62]. The effect of geometric and material nonlinearities of the mounting on transmissibility of simple system is studied in references [4, 63].

1.2.5 Analysis of an unidirectional multidegrees of freedom model

Two stages or compound mounting can provide especially low values of transmissibility at high frequencies, if added mass can be tolerated. Application of this system and design of one mount is shown in references [1,2]. Complete analysis

of compound stage mounting with rigid and nonrigid foundations and frequency dependent properties of rubber is given.

Criteria for synthesis of multidegrees of freedom nondissipative (no damping), low pass vibration isolators with low transmissibility over a wide frequency range are established by Munjal [64].

An analytical procedure for the evaluation of transmissibility of an n degrees of freedom viscoelastic antivibration mounting is developed in reference [65]. Transmissibility is determined by means of a class of polynomials depending on the frequency of the exciting force and the mechanical properties of the system. Generalisation of unequal masses and dissimilar elements can directly be obtained. Later analysis of this system for transient excitation also is done in reference [66].

1.2.6 Analysis of two or multipoint excitation model

Analysis of a rigid body supported by undamped isolators of constant stiffness has been done for one and two planes of symmetry [18,20].

Pinnington [45] has derived the expressions for vibration power flow when a motor is supported by four isolators on a flexible seating system. A detailed dynamic analysis of a heavily damped foundation structure designed for the multidirectional isolation of a diesel engine has

been presented in reference [46]. Using modal analysis a modelling technique for the prediction of vibration transmission to a structure is presented in reference [48]. This can be extended to tackle multidirectional vibration isolation problems of machinery mounted on nonrigid foundations.

General classification of machinery and formulation of typical features of a dynamic vibro isolation system of machinery, criteria of effective isolation for main groups of machinery are given in reference [51]. Design of isolators complying with these criteria are also described.

Analysis of isolation of rigid body supported by rubber isolators whose stiffness and damping are frequency dependent and when the foundation has finite impedance is not presented anywhere in the literature.

1.2.7 Wave effects in isolators

Wave effects may be observed at high frequencies when the mount dimensions become comparable with multiples of the half-wave lengths of elastic waves travelling through the mounting. Alternatively, wave effects may be thought of as occurring when the elasticity and the distributed mass of the rubber mounting interact at high frequencies. The geometry of rubber components of antivibration mountings is frequently complex, which makes precise theoretical calculations of transmissibility difficult at high frequencies. A guide to the character of wave effects in antivibration

mountings has been obtained, however, by considering the transmissibility of "mountings" that obey the simple wave equation for the longitudinal vibration of a rod of uniform cross section. This analysis is based on long rod theory where lateral dimensions of the rods remain small compared with the wavelength. Love theory uses corrected wave equation for analysing wave effects in cylindrical rod like mount of significant lateral dimensions. A parametric study of the above analysis is given in reference [1].

In his review paper [2] Snowdon has explained the application of the four pole parameter analysis which enables a general account to be taken of wave effects in antivibration mountings and lack of rigidity in the foundation and mounted item.

A cylinder like rubber mount is considered as a lumped parameter model and the interaction between distributed mass and elasticity at high frequencies is considered in reference [67]. Analysis of wave effects of an isolator consisting of n equal masses and n intermediate distributed elastic mounts is presented in reference [68]. This analysis is based on four pole parameters.

Disregarding the longitudinal wave motion at high frequencies may lead to an apparent increase of the dynamic modulus and to an apparent decrease of the loss factor. This is shown in reference [69] when the transfer function method

is used to investigate the dynamic characteristics as a function of frequency for a mass loaded spring type specimen.

1.2.8 Experimental methods to measure transmissibility and power flow

Direct measurements of transmissibility which make use of an apparatus built to simulate the simple mounting system is explained in references [1,2]. This work also include many references for this system and for the experimental determination of transmissibility across a compound mounting system. A indirect measurement of transmissibility based on four pole parameter technique is explained in reference [2]. The advantage is that the antivibration mount can be held by the support block in the manner likely to be encountered "in service". Driving point impedance and quasitransfer impedance of antivibration mount under significant static loads can be measured.

Direct measurements of transmissibility have been reported in references [4, 63] under more realistic amplitude and frequency ranges using a single degree of freedom model when the rubber is in simple shear. The effect of nonlinearity on transmissibility is also studied.

Experimental measurements of power transmission are presented by Pinnington and White [38]. For single point excitation model, power input at the top of the isolator and

the power transmitted to the "infinite" and finite seatings are measured. The following two methods of measuring power are described.

(i) Power input at a point on the structure can be measured by multiplying the imaginary component of the point apparent mass by the acceleration spectral density at that point.

(ii) Power transmission through a spring like isolator can be measured by using the isolator dynamic stiffness and the cross spectrum of the acceleration signals above and below the isolator.

An experimental procedure is described in reference [70] to measure the vibrational power input to a structure when subjected to multiple forces. The validity of the method is examined and used to measure the power absorbed by a plate.

The isolator transfer apparent mass method and seating point apparent mass method are explained in detail to measure the vibration power transmitted from a motor supported by four isolators to a seating structure in reference [45].

1.3 Objectives and Scope of the Present Work

In the present study, vibration isolation of machines from finite-impedance foundations through bonded rubber isolators is of primary interest. An attempt is made to bring out the effects of the finite impedance of the foundation

on the performance of the vibration isolation system. Expressions for response ratio, vibration power flow are derived for various cases. The concept of parallel mounting is examined by introducing the frequency dependence of dynamic modulus and loss factor of rubber like materials.

So far the analysis of a unidirectional multi degree-of-freedom vibration isolation system taking into account the wave effects in the isolator has been carried out by the so called "four pole parameter" technique [68]. Expressions for the transmissibility thus derived become cumbersome for a periodic isolator with a large number of elements. In this work, a mobility technique is used along with the concept of periodicity. This renders in simple expressions for the transmissibility where the computational effort is independent of the number of elements.

The analysis of isolation of a body with unidirectional motion is extended to that of a rigid body with both heave and roll modes. Wave effects are also incorporated in the analysis. This analysis can be easily extended to more complex systems such as with one plane of symmetry, multipoint excitation model etc.

Throughout the analysis, the foundation is modelled as nonresonating, i.e. either finite and rigid (mass like) or infinite and non-rigid. The foundations are described in terms of the relevant driving point mobilities. The foundation flexibility in only one direction is considered. The machine to be isolated (source) is treated as a rigid mass. The force

excitation is considered to be harmonic in nature. Random and transient excitations are not considered.

The system under consideration is linear and no attempt is made to study the effects of nonlinearities. Though the properties of rubber depend on many parameters, only the frequency dependence is considered in the present analysis.

Only force source idealisation is used, but it can be easily extended to velocity source idealisation. The rotational mobilities of the isolators are ignored.

This analysis is concerned mainly towards correct modelling and developing general guidelines for designing a vibration isolation system. For practical applications, many other aspects such as constraints of geometry, stresses, amplitudes of vibrations etc. have to be considered and various dimensions of the mountings may be selected to satisfy all the constraints.

CHAPTER II

CHARACTERISTICS OF ISOLATORS AND FOUNDATIONS

2.1 Introduction

In this chapter, the dynamic properties of rubber like materials with reference to vibration isolation are discussed. Advantages of bonded rubber springs over unbonded springs are pointed out. Approximate stress-strain formulae for bonded rubber springs when subjected to various loading patterns are studied. The concept of parallel mounting and its advantage is explained.

Modelling of finite-impedance foundations in terms of infinitely long structures is explained and the salient features of this modelling are studied. The concepts of power flow are introduced. Various expressions used to evaluate the performance of isolator are explained and correlated.

2.2 Isolator Materials and Constructions

2.2.1 Dynamic properties of rubber like materials

The dynamic properties of rubber like materials experiencing sinusoidal vibration are accounted for by writing the elastic moduli that govern the vibration as complex quantities. The Young's modulus and the shear modulus are most generally written

$$E^*_{\omega,\theta} = E_{\omega,\theta} (1 + j \, \delta_{E_{\omega,\theta}})$$

and

$$G^*_{\omega,\theta} = G_{\omega,\theta} (1 + j \, \delta_{G_{\omega,\theta}})$$

Here, the asterisk denote complex quantities and $j = \sqrt{-1}$. The real parts of the complex moduli are called dynamic moduli and the ratios of imaginary part to the real part are called damping factor or loss factor. The subscripts ω and θ indicate that the dynamic moduli and damping factor are, in general, functions of both angular frequency ω and temperature θ.

The concept of complex modulus means that the harmonic strain lags the stress in a rubber like material by an angle, the tangent of which is the damping factor $\delta_{E_{\omega,\theta}}$ or $\delta_{G_{\omega,\theta}}$.

For rubber like materials, the complex shear and Young's moduli exhibit the same frequency and temperature dependence, that is to say

$$E_{\omega,\theta} = 3 \, G_{\omega,\theta}$$

and

$$\delta_{E_{\omega,\theta}} = \delta_{G_{\omega,\theta}}$$

The dynamic moduli are found to increase with increasing frequency (at a given temperature) and decreasing temperature (at a given frequency). The transition frequency ω_t and

temperature Θ_t refer to the transition of rubber like materials at sufficiently high frequencies or sufficiently low temperatures to an inextensible or glass like state. The elastic moduli (real part) become so large that the characteristic resilience of the material is no longer apparent. At the socalled rubber-to-glass transition, the damping factor passes through a maximum value that lies approximately in the frequency or temperature range through which the real part of the elastic moduli increases most rapidly.

The parallel combination of spring and viscous dashpot is frequently discussed in the literature concerned with anti-vibration mounting. However, it is important to recognize that the dynamic properties of the combination poorly represent those of rubber like materials. The three element combination of two springs and one dashpot also fails to provide a satisfactory representation of rubber like materials [1,2].

2.2.2 Bonded rubber springs

Elastomeric isolators (rubber springs) may be designed in both bonded and unbonded configurations. In the bonded isolator, metal inserts are bonded to the rubber on all load-carrying surfaces. In the unbonded or semibonded isolator, the rubber load-bearing surface rests directly on the support structure. Bonded parts are generally preferred even though they are costly since they may be more highly stressed for a given deflection. With higher stresses they provide higher spring constants

and higher elastic energy storage capacity.

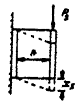
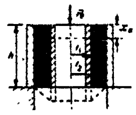
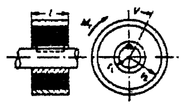
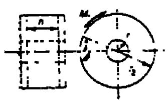
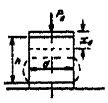
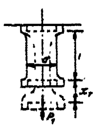
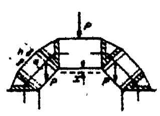
Bonded isolators can be designed to provide proper load distribution in shear, compression, tension or combination loading. A more uniform stress distribution in the rubber is obtained by bonding inserts on all the load bearing rubber surfaces. In contrast, unbonded parts usually fail to distribute the load uniformly and thus resulting in local areas of stress concentration in the rubber body which shorten the life of the isolator.

A significant difference between bonded and unbonded rubber isolators relates to how they behave under load. Load-deflection characteristics of unbonded isolator depends on the maintenance of friction at the rubber-support structure interface. When consistent load-deflection characteristics are required for the life of the equipment, bonded isolators should be used [18].

Rubber springs may be loaded in various modes, namely, tension, compression, simple shear, axial shear, rotary shear, torsional shear and bending stress. The loading may occur in any one or a combination of these several modes. The expressions for the stiffness of such bonded rubber springs can be obtained using simple stress-strain (Hooke's law) relation. The socalled spring equations of bonded rubber springs are listed in Table (2.2.2.1).

Table 2.2.2-1

(Produced from reference [19])

Loading	*Type of spring*	*Spring equation*	*Applicability limit*
Rubber spring mounting in simple shear		$P_s = x_s \frac{GA}{R}$	$x_s = 0.35h$
Sleeved rubber spring in axial shear		$P_a = x_a \frac{2\pi h G}{\ln(r_2/r_1)}$	$x_a = 0.35(r_2 - r_1)$
Sleeved rubber spring in rotary shear		$M_r = \vartheta \frac{4\pi L G \times 10^3}{(1/r_1^2 - 1/r_2^2)}$	$\vartheta = 40°$
Rubber disc spring in torsion		$M_t = \vartheta \frac{\pi G(r_2^4 - r_1^4)}{10^3\, 2h}$	$\vartheta = 20°$
Rubber spring pad in compression		$P_c = x_c \frac{d^2 \pi E_a}{4h}$	$x_c = 0.2h$ $\Sigma(d, b, w) > 4h_\Sigma$
Rubber spring rod in tension		$P_t = x_t \frac{d^2 \pi E}{4L}$ $E = 3G$	$x_t = 0.40l$
Rubber spring pads under shear-compression		$P_\Sigma = \frac{2xA}{h}(G \sin^2 \alpha + E_r \cos^2 \alpha)$	dependent on α

The spring stiffness is dependent on the property of the rubber used and on the geometry of the isolator. For dynamic shear loading, the spring stiffness is expressed in the form

$$K^{*} = K\, G^{*}_{\omega}$$

where G^{*}_{ω} is complex modulus given by $G^{*}_{\omega} = G_{\omega}(1 + j\, \delta_{G_{\omega}})$ whose temperature dependence is ignored in the present analysis and K is a parameter which depends on geometry of the isolator and has dimension of length. Similarly, for dynamic loading of other patterns the spring stiffness can be expressed in terms of a geometric factor and the corresponding complex moduli.

2.2.3 Parallel mounting

An isolator, to be effective at all frequencies in a single degreeof-freedom system, must satisfy the following requirements:

(i) The stiffness of the isolator should be low so as to make $\omega_o \ll \omega$, i.e., $\omega/\omega_o \gg 1$ such that the transmissibility is much less than unity; where ω is the exciting frequency and ω_o is the resonance frequency of the system.

(ii) The damping present in the isolator should not be too low in order to limit the resonance transmissibility.

(iii) The damping present in the isolator should not be too high in order to have a good high-frequency isolation characteristic.

Making a very soft isolator (i.e. with a low value of ω_o is practically difficult due to the problems of side wise buckling and large static deflection.

The requirements numbered (ii) and (iii) are obviously contradictory to each other and a compromise solution is achieved through what is known as a parallel mounting [1]. Such a mount is constructed by combining natural and synthetic rubbers. Both the stiffness and damping of the combination may be frequency dependent.

Natural rubber possesses almost constant stiffness and low damping whereas the stiffnesses of high damping synthetic rubbers vary almost linearly with the frequency. These two materials can be used in a judicious combination so that (i) the stiffness does not vary too sharply, (ii) the damping near resonance is not too low and (iii) the damping in the high frequency region is not too high. Figure 2.2.3-1 shows such a parallel mounting working in shear where subscripts 1, 2 refer, respectively to the low and high damping rubbers; α and G^*_ω refer respectively to the cross-sectional areas and the complex shear moduli. The resultant complex shear modulus of the mount material is then given by

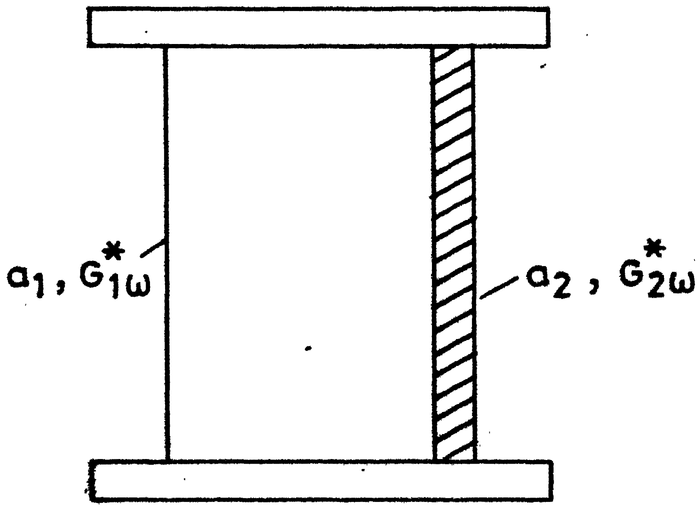


Fig. 2.2.3-1

Parallel Mounting

$$G^*_{P_\omega} = \frac{a_1}{(a_1 + a_2)} G^*_{1_\omega} + \frac{a_2}{(a_1 + a_2)} G^*_{2_\omega} = \frac{G^*_{1_\omega} + a' G^*_{2_\omega}}{(1 + a')}$$

where $a' = a_2/a_1$

It follows that

$$G_{P_\omega} = \frac{(G_{1_\omega} + a' G_{2_\omega})}{(1 + a')} \qquad (2.2.3\text{-}1)$$

and

$$\Delta P_\omega = \frac{(G_{1_\omega} \delta_{1G_\omega} + a' G_{2_\omega} \delta_{2G_\omega})}{(G_{1_\omega} + a' G_{2_\omega})} \qquad (2.2.3\text{-}2)$$

G_{P_ω} and ΔP_ω are dynamic modulus and damping factor of the parallel mounting respectively.

Frequency dependence of dynamic modulus and damping factor possessed by a parallel combination of natural rubber and polyvinyl butyl resin for different values of a' is shown in reference [1].

The designer has to choose the two materials and the value of a' for obtaining the suitable compromise, i.e. the proper values of G_{P_ω} and ΔP_ω .

Unless otherwise mentioned all the systems in this work are analysed considering the properties of the previously mentioned parallel combination.

2.3 Characterisation of Foundations

2.3.1 Foundation of finite impedances

Typical built-up structures such as buildings and ships machinery foundations often consist of beams, plates and beam stiffened plates. In the present work, the characteristics of these foundations, their power transmission mechanisms are considered. Power flow formulations and the approximate structural frequency response for these foundation structures are presented in references [39,40].

At low frequencies, discontinuities at the boundaries of these structural elements cause travelling waves to be reflected and resonances to occur. At higher frequencies, resonant behaviour is less apparent because the travelling waves are transmitted through the boundaries and power is radiated or absorbe in remote parts of the structure.

A finite structure may be approximated by an equivalent structure of infinite extent with no reflecting devices. It may be assumed that the waves propagating away from the source are attenuated by damping or radiation and are not reflected back to form standing waves. Alternatively this is equivalent to assuming that there are many modes of vibration contributing to the motion at any one frequency without one mode being dominant.

At low frequencies, where the resonances are well separated the above approximation is less accurate but is still valuable sinc

it gives the average level of the response. Thus the motion of a machinery foundation may be approximated by considering it to be of infinite extent.

A number of typical foundations such as beams, plates and beam stiffened plates have, therefore, been analysed in references [39,40] and wave propagation in these types of structure has been studied. Force and/or torque excited foundations are considered for power flow calculations.

2.3.2 Power flow concepts

Power is the rate at which work is done and is given by the relationship $P_i = \tilde{F}_i \tilde{V}_i$ (2.3.2-1)

where $\tilde{F}_i$ and $\tilde{V}_i$ are the instantaneous values of force and velocity at a point. When the power flows through an area, it is necessary to consider it as an intensity and therefore, the force $\tilde{F}_i$ is considered as the stress. With a vibrating structure the net flow of power is more important than the instantaneous value and when both the force and velocity are harmonic, this is given by

$$P = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \tilde{F}_i \tilde{V}_i \, dt \qquad (2.3.2\text{-}2)$$

where ω is the frequency of vibration. If the force and velocity are written as $\tilde{F}_i = F e^{j\omega t}$ and $\tilde{V}_i = V e^{j\omega t}$ where F and V are complex and may thus include a relative phase angle. Then

$$P = \frac{1}{2} |V| \; |F| \; \cos \phi$$

or,

$$P = \frac{1}{2} \operatorname{Re} \{F V^*\} = \frac{1}{2} \operatorname{Re} \{F^* V\} = \frac{1}{2} [\operatorname{Re} \{F\} \operatorname{Re} \{V\} + \operatorname{Im} \{F\} \operatorname{Im} \{V\}]$$

(2.3.2-3)

where ϕ is the relative phase angle and * denotes the complex conjugate, The ratio of the complex harmonic velocity to the complex harmonic force, is the mobility and this quantity is a property of the structure alone. One may substitute, therefore, for either the force or velocity to give

$$P = \frac{1}{2} |F|^2 \operatorname{Re}\{\beta\} = \frac{1}{2} |V|^2 \operatorname{Re}\{\beta\} / |\beta|^2 \qquad (2.3.2\text{-}4)$$

where

$$\beta = V/F$$

A convenient method of analysing infinite structures is to use spatial Fourier transforms in conjunction with contour integration techniques. These procedures are explained in reference [39]. The analysis is based on considering the torsional, longitudinal and flexural wave motion.

The driving point mobilities and expressions for the input power flow with torque and force excitation are listed in Table 2.3.2-1.

2.3.3 Comparison between finite and infinite structures

The formulae given in Table [2.3.2-1] apply to infinite structures in which no resonance can occur. Due to reflections from discontinuities, any finite structure will exhibit resonances which would not be apparent in the response of an infinite structure. In order to simplify forced vibration calculations in a practical case, response estimates may be made by using the mobility and power flow formulae for infinite structures. A comparison between the monotonic characteristic derived from the simple, infinite formulation and the strongly peaked frequency response of the finite structure should be made.

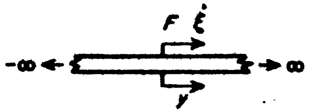
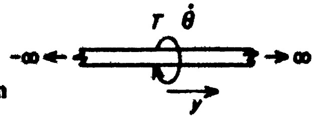
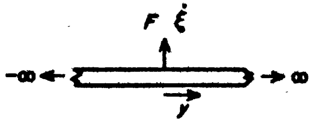
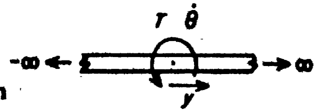
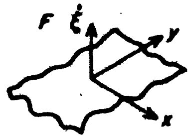
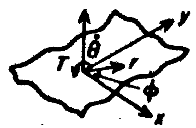
The magnitude of the vibration amplitudes at resonances are controlled by the damping of the structure and generally the largest response will occur at the first resonance. Thus when a finite structure is being represented by an equivalent infinite structure the largest error that will occur will be at a resonance frequency. The mobility at the driving point of any finite structure may be written as

$$\frac{V}{F} = -j\omega \sum_{n} \frac{[\psi_n]^2}{\omega_n^2 \cdot (1-j\eta) - \omega^2} \qquad (2.3.3\text{-}1)$$

where ω_n is the resonance frequency, η is hysteritic loss factor and ψ_n is the amplitude of the mode shape at the driving point, n being the mode number. At a low frequency resonance, for light damping, the contribution of offresonance terms is negligibl

Table 2.3.2-1
(produced from reference [39])

*Properties of infinite system (notes: *, torque applied about axis parallel to l_2; time dependence of form $e^{i\omega t}$ assumed)*

System		Driving point mobility	Power flow into system (P_s) force or torque source	Power flow into system; velocity or angular velocity source
Beam longitudinal wave motion; force excitation		$\frac{\xi}{F} = \frac{1}{2A\sqrt{E\rho}}$	$P_s = \frac{\lvert F\rvert^2}{4A\sqrt{E\rho}}$	$P_s = 4\lvert\dot{\xi}\rvert^2 A\sqrt{E\rho}$
Beam torsional wave motion; torque excitation		$\frac{\dot{\theta}}{T} = \frac{1}{2\sqrt{GQJ}}$	$P_s = \frac{\lvert T\rvert^2}{4\sqrt{GQJ}}$	$P_s = 4\lvert\dot{\theta}\rvert^2\sqrt{GQJ}$
Beam flexural wave motion; force excitation		$\frac{\xi}{F} = \frac{(1-i)}{4A\rho\sqrt{\omega}}\left(\frac{A\rho}{EI}\right)^{1/4}$	$P_s = \frac{\lvert F\rvert^2}{8A\rho\sqrt{\omega}}\left(\frac{A\rho}{EI}\right)^{1/4}$	$P_s = \lvert\dot{\xi}\rvert^2 A\rho\sqrt{\omega}\left(\frac{EI}{\rho A}\right)^{1/4}$
Beam flexural wave motion; torque excitation		$\frac{\dot{\theta}}{T} = \frac{(1+i)\sqrt{\omega}}{4EI}\left(\frac{EI}{\rho A}\right)^{1/4}$	$P_s = \frac{\lvert T\rvert^2\sqrt{\omega}}{8EI}\left(\frac{EI}{\rho A}\right)^{1/4}$	$P_s = \frac{\lvert\dot{\theta}\rvert EI}{\sqrt{\omega}}\left(\frac{\rho A}{EI}\right)^{1/4}$
Plate flexural wave motion; force excitation		$\frac{\xi}{F} = \frac{1}{8\sqrt{B_p\rho h}}$	$P_s = \frac{\lvert F\rvert^2}{16\sqrt{B_p\rho h}}$	$P_s = 4\lvert\dot{\xi}\rvert^2\sqrt{B_p\rho h}$
Plate flexural wave motion; torque excitation		$\frac{\dot{\theta}}{T} = \frac{\omega}{8B_p(1+L)} \times\left[1-\frac{i4}{\pi}\ln ka + \frac{i8L}{\pi(1-\nu)}\left(\frac{h}{\pi a}\right)^2\right]$	$P_s = \frac{\omega\lvert T\rvert^2}{16B_p(1+L)}$	$P_s = \frac{4\lvert\dot{\theta}\rvert^2 B_p(1+L)}{\dot{\omega}\left\{1+\left[\frac{4}{\pi}\ln ka - \frac{8L}{\pi(1-\nu)}\left(\frac{h}{\pi a}\right)^2\right]^2\right\}}$

B_P - Bending stiffness of plate, GQ - Torsional stiffness,

a - Radius of disc over which torque applied to plate acts,

L - Parameter which tends to unity for large a/h,

J - Polar moment of inertia per unit length.

Table 2.3.3-1

(produced from reference [39])

System	Onset of infinite behaviour	Largest point mobility of finite system	Ratio of finite system maximum to infinite system	Wavenumber (k)	Displacement of structure
Beam longitudinal wave motion; force excitation	$\omega > \frac{\pi}{\eta l}\sqrt{\left(\frac{E}{\rho}\right)}$	$\beta_l = \frac{2}{\pi A \eta \sqrt{E\rho}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4}{\pi\eta}$	$k = \omega\sqrt{\left(\frac{\rho}{E}\right)}$	$\xi(y) = \frac{-\mathrm{i}F\,\mathrm{e}^{-\mathrm{i}ky}}{2\omega A\sqrt{E\rho}}$
Beam torsional wave motion; torque excitation	$\omega > \frac{\pi}{\eta l}\sqrt{\left(\frac{GQ}{J}\right)}$	$\beta_l = \frac{2}{\pi\eta\sqrt{GQJ}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4}{\pi\eta}$	$k = \omega\sqrt{\left(\frac{J}{GQ}\right)}$	$\theta(y) = \frac{-\mathrm{i}T\,\mathrm{e}^{-\mathrm{i}ky}}{2\omega\sqrt{GQJ}}$
Beam flexural wave motion; force excitation	$\sqrt{\omega} > \frac{4\pi}{\eta l}\left(\frac{EI}{\rho A}\right)^{1/4}$	$\beta_l = \frac{2l}{\pi^2\eta\sqrt{\rho AEI}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4\sqrt{2}}{\pi\eta}$	$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{1/4}$	$\xi(y) = \frac{-\mathrm{i}F}{4EIk^3}[\mathrm{e}^{-\mathrm{i}ky} - \mathrm{i}\,\mathrm{e}^{-ky}]$
Beam flexural wave motion; torque excitation	$\sqrt{\omega} > \frac{4\pi}{\eta l}\left(\frac{EI}{\rho A}\right)^{1/4}$	$\beta_l = \frac{2}{l\eta\sqrt{\rho AEI}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{2\sqrt{2}}{\pi\eta}$	$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{1/4}$	$\xi(y) = \frac{T}{4EIk^2}[\mathrm{e}^{-\mathrm{i}ky} - \mathrm{e}^{-ky}]$
Plate flexural wave motion; force excitation	$\omega > \frac{8}{\eta l_1 l_2}\sqrt{\left(\frac{B_p}{\rho h}\right)}$	$\beta_l = \frac{4l_1 l_2}{\pi^2\eta\sqrt{\rho h B_p}(l_1^2 + l_2^2)}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{32 l_1 l_2}{\pi^2\eta(l_1^2 + l_2^2)}$	$k = \sqrt{\omega}\left(\frac{\rho h}{B_p}\right)^{1/4}$	**Valid in far field only** $\xi(r,\phi) = \frac{-\mathrm{i}F}{8B_p k^2}\sqrt{\left(\frac{2}{rk\pi}\right)}\mathrm{e}^{-\mathrm{i}(rk-\pi/4)}$
Plate flexural wave motion; torque excitation	$\omega > \frac{8}{\eta l_1 l_2}\sqrt{\left(\frac{B_p}{\rho h}\right)}$	$\beta_l = \frac{16 l_2}{\eta\sqrt{\rho h B_p}\, l_1(2l_2^2 + l_1^2)}$ *		$k = \sqrt{\omega}\left(\frac{\rho h}{B_p}\right)^{1/4}$	**Valid in far field only** $(r,\phi) = \frac{T}{8B_p k}\sqrt{\left(\frac{2}{rk\pi}\right)}\mathrm{e}^{-(rk-\pi/4)}\sin\phi$

compared to the magnitude of the term at resonance. Thus the driving point mobility of a finite structure at resonance may be written as

$$\frac{V}{F} = [\Psi_n]^2 / \omega_n \eta \qquad (2.3.3\text{-}2)$$

if there is a wide frequency spacing between resonances. It may be seen that the mobility value in general is largest for the lowest value of resonance frequency, if Ψ_n does not vary significantly for different resonances.

The largest peaks in the mobility spectra of finite beams and plates have been calculated in reference [39]. These peak mobility values represent a worst case if a finite structure is modelled as being infinite. Table [2.3.3-1] contains a list of the peak mobilities and also a list of the ratios of the peak point mobility of the finite structure to the point mobility of the infinite structure, written in modulus form rather than as a complex quantity. The ratio of the mobility of the finite structure at resonance to the mobility of the infinite case takes a particularly simple form, in most cases being inversely proportional to loss factor.

It can be concluded that several types of foundations can be analysed based on the assumption that they are infinite in extent, so long as there are no significant reflections from boundaries within the foundations. The moduli of driving point mobilities of these foundations may be represented by straight lines when plotted against frequency on log log scales.

2.3 Expressions for Evaluating Performance of an Isolator

Several quantities, as mentionedin the literature review earlier, are used as pertinent indices to measure the performance of an isolator. Some of them are explained below.

(1) Transmissibility (T)

Transmissibility T, across a system is defined as the ratio of magnitude of the force transmitted to the foundation to magnitude of the exciting force acting on the system. Similarly, velocity and displacement transmissibilities can be defined. Force, velocity and displacement transmissibilities are identical for a simple system. Transmissibility does not give the correct picture when the foundation is nonrigid.

(2) Response ratio (R)

Response ratio R, is defined as the ratio of magnitude of the forces transmitted to the foundation with and without the isolator. This is identical to transmissibility, if the foundation is of infinite impedance. For finite impedance foundation, this index provides a better description of the isolator performance.

(3) Isolation effectiveness (IE)

This is nothing but the reciprocal of the response ratio R and better isolator implies higher values of IE where as

T&R decrease to signify better isolation.

(4) Power flow to foundation (P)

$$P = \frac{1}{2} \text{ Re } \{\beta\} \; |\tilde{F}_Z|^2 \; = \; Q_f \; (\omega) \; |F|^2$$

where β is the foundation mobility and $\tilde{F}_Z$ is the force transmitted to foundation, F is the excitation force.

$Q_f(\omega)$ is purely real and depends only on properties of the isolator, machine and foundation. $Q_f(\omega)$ is used as an index of vibration isolation. This is claimed as a better representation as it includes forces as well velocities in a single concept and takes care in case of finite seating area.

(5) Power flow ratio (PF)

Power flow ratio PF, is defined as the ratio of average power transmitted to the foundation with and without the isolator. This is nothing but twice the value of response ratio when plotted versus frequency on a log-log scale.

All the above quantities are generally plotted on log-log scale with frequency ratio on the X-axis.

CHAPTER III

SINGLE POINT EXCITATION MODEL

3.1 Introduction

In this chapter, the machine to be isolated is treated as a rigid mass moving in one direction, and coupled by only a single isolator to a infinite impedance or finite impedance seating as indicated in Figs. 3.1-1 to 3.1.-3 . Expressions for the transmissibility, response ratio, vibrational power flow, defined in the previous chapter, are derived to study the effectiveness of the isolator. A parametric study is done to evaluate the effects of different types of nonrigid seating structures on vibration isolation. Approximate expressions for the point mobility of equivalent infinite seating structures are used. The effects of frequency dependence of the isolator material properties is shown. Systems with some kind of periodicity are analysed using a mobility technique. Using this analysis periodic isolation systems and periodic isolators are investigated.

3.2 Foundations of Infinite Impedance

The simple mounting system shown in Figs.3.1-1 and 3.1-2 where an element of mass M is supported by a linear

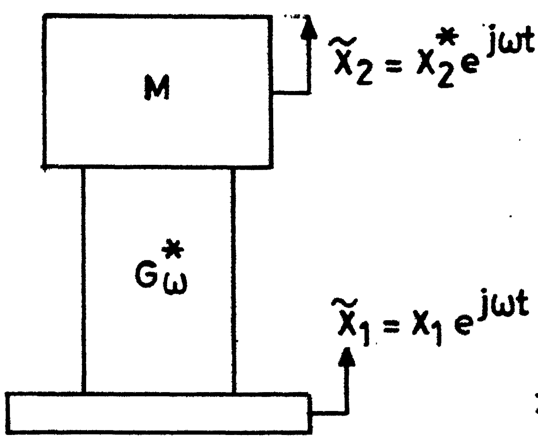


Fig. 3.1-1

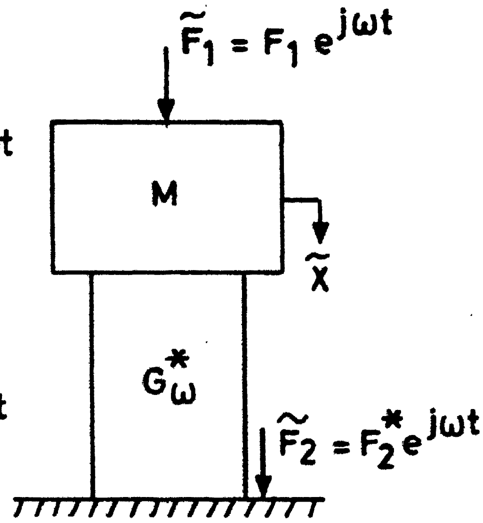


Fig. 3.1-2

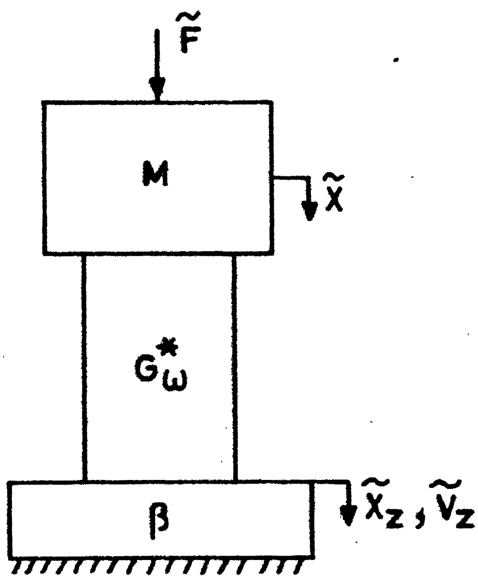


Fig. 3.1-3

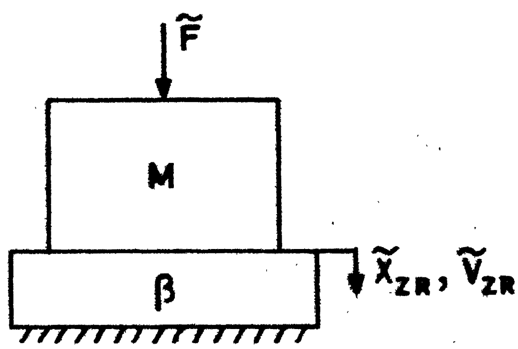


Fig. 3.1-4

Simple Mounting

rubber like material on a foundation. The complex shear modulus of the isolator is given by $G^*_{\omega,\theta}$. Here and subsequently, it is assumed that the temperature remains constant so that $G^*_{\omega,\theta}$ may be written as

$$G^*_{\omega} = G_{\omega} (1 + j \ \delta \ G_{\omega}) \qquad (3.2\text{-}1)$$

The mounted item M is assumed to be supported at its center of gravity and to vibrate only in the vertical direction. It is excited by a sinusoidally varying ground displacement $\tilde{x}_1$ as in Fig. 3.1-1 or by a sinusoidally varying force $\tilde{F}_1$ as in Fig. 3.1-2. If the transmissibility T across the first system is defined as the magnitude of displacement ratio $|\tilde{x}_2/\tilde{x}_1|$, and if the transmissibility across the latter is defined as the magnitude of the force ratio $|\tilde{F}_2 / \tilde{F}_1|$ then, at any one frequency,

$$T = |\tilde{x}_2 / \tilde{x}_1| = |\tilde{F}_2 / \tilde{F}_1| \qquad (3.2\text{-}2)$$

where $\tilde{x}_2$ is the displacement of M in Fig. 3.1-1 and
$\tilde{F}_2$ is the force transmitted to the ideally rigid foundation in Fig. 3.1-2.

Thus, the results of a single calculation or measurement o transmissibility have dual significance.

The equation of motion for the mass M of the system in Fig. 3.1-1 may be written as

$$M\left(\frac{d^2 \tilde{x}_2}{dt^2}\right) = -\omega^2 M \tilde{x}_2 = K G_\omega^* (\tilde{x}_1 - \tilde{x}_2) \qquad (3.2\text{-}3)$$

where K is a factor solely governed by the geometry of the isolator. It has the dimensions of length. For rubbermount of cross sectional area 'a' and length ℓ ,

$$K = 3\ (a/\ell)\ (1 + B\ S^2) \qquad (3.2\text{-}4)$$

where B is a numerical constant and S is a dimensionless shape factor defined as the ratio of the area of one loaded surface to the total force free area of the rubber. More simply, if the rubber element is used directly in shear rather than as drawn in Fig. 3.1-1, K = (a/ℓ)

Since $\tilde{x}_1 = x_1\ e^{j\omega t}$ and $\tilde{x}_2 = x_2^*\ e^{j\omega t}$, eqn. (3.2-3) can be written as

$$-\omega^2 M\ x_2^*\ e^{j\omega t} = K\ G_\omega^*\ (x_1 - x_2^*)\ e^{j\omega t}$$

or,

$$\frac{x_2^*}{x_1} = \frac{K\ G^*}{(KG_\omega^* - \omega^2 M)} = \frac{(1 + j\ \delta G_\omega)}{[1 - (\omega^2 M/KG_\omega) + j\delta G_\omega]} \qquad (3.2\text{-}5)$$

The natural frequency ω_o of the simple system is defined as the value of ω for which, in the absence of damping, x_2 becomes infinitely large. Thus if $G_\omega = G_o$ at this particular frequency, then

$$\omega_o^2 = KG_o/M \tag{3.2-6}$$

The transmissibility across the simple mounting system is obtaine from eqns. (3.2-2) and (3.2-5), as

$$T = (1+ \delta_{G_\omega}^2)^{1/2} / \{[1 -(\omega / \omega_o)^2(G_o/G_\omega)]^2 + \delta_{G_\omega}^2\}^{1/2} \tag{3.2-7}$$

In the present work the isolator is modelled as a parallel rubber mounting discussed in section 2.2.3. The parallel combination of a low-damping natural rubber and a high-damping synthetic rubber is considered. The properties of these material: are taken from reference [1]. Using a curve fitting procedure, the dynamic modulus and damping factor of the isolator are finally represented in the form

$$G_\omega = G_{P_\omega} = a_1 \omega^{b_1} \tag{3.2-8}$$

where ω is expression in (Hz)

$$\Delta P_\omega = \delta_{G_\omega} \tag{3.2-9}$$

For different values of a' in eqns. (2.2.3-1) and (2.2.3-2), the values of all the parameters involved in eqns.(3.2-8) and

(3.2-9) are evaluated and listed in Table 3.2-1. Using eqns. (3.2-8) and (3.2-9) in (3.2-7), one gets

$$T = (1 + \delta^2_{G_\omega})^{1/2} / \{[1 - (\omega/\omega_o)^{2-b}]^2 + \delta^2_{G_\omega}\}^{1/2} \tag{3.2-10}$$

This expression of T is evaluated for three types of rubbers namely low damping, high damping and a parallel combination of these two. Corresponding plots of transmissibility and frequency ratio on log-log scale are shown in Fig. 3.2-1. The graph of parallel combination rubber refers to the case when $a' = 0.1$. The advantage of parallel combination as opposed to either high or low damping rubber is evident from the figure. The cross-sectional areas of the low and high damping rubbers (i.e. the value of a') are chosen so that, at frequencies near to the natural frequency (ω_o) of the mounting system, the low damping rubber possesses the greater stiffness. At frequencies immediately above ω_o, therefore, the transmissibility of the parallel mounting decreases rapidly with frequency because the contribution of the high damping rubber to the stiffness of the mounting is relatively small. The transmissibility of the mountin at resonance is less than that of the low damping rubber, since the damping factor of the mounting is greater than that of the low damping rubber alone.

Table 3.2-1 : Properties of Various Parallel Combinations of Low and High Damping Rubbers

Area ratio a'	a_1 ($\times 10^6$ N/m^2)	b_1	δG_ω
0 (low damping natural rubber)	6.1	0.05	0.1
0.1	6.1653	0.145	0.3096
0.2	6.256	0.192	0.3953
0.3	6.371	0.222	0.4419
1.0	6.9709	0.302	0.5391
∞ (high damping synthetic rubber)	7.0	0.4	0.7

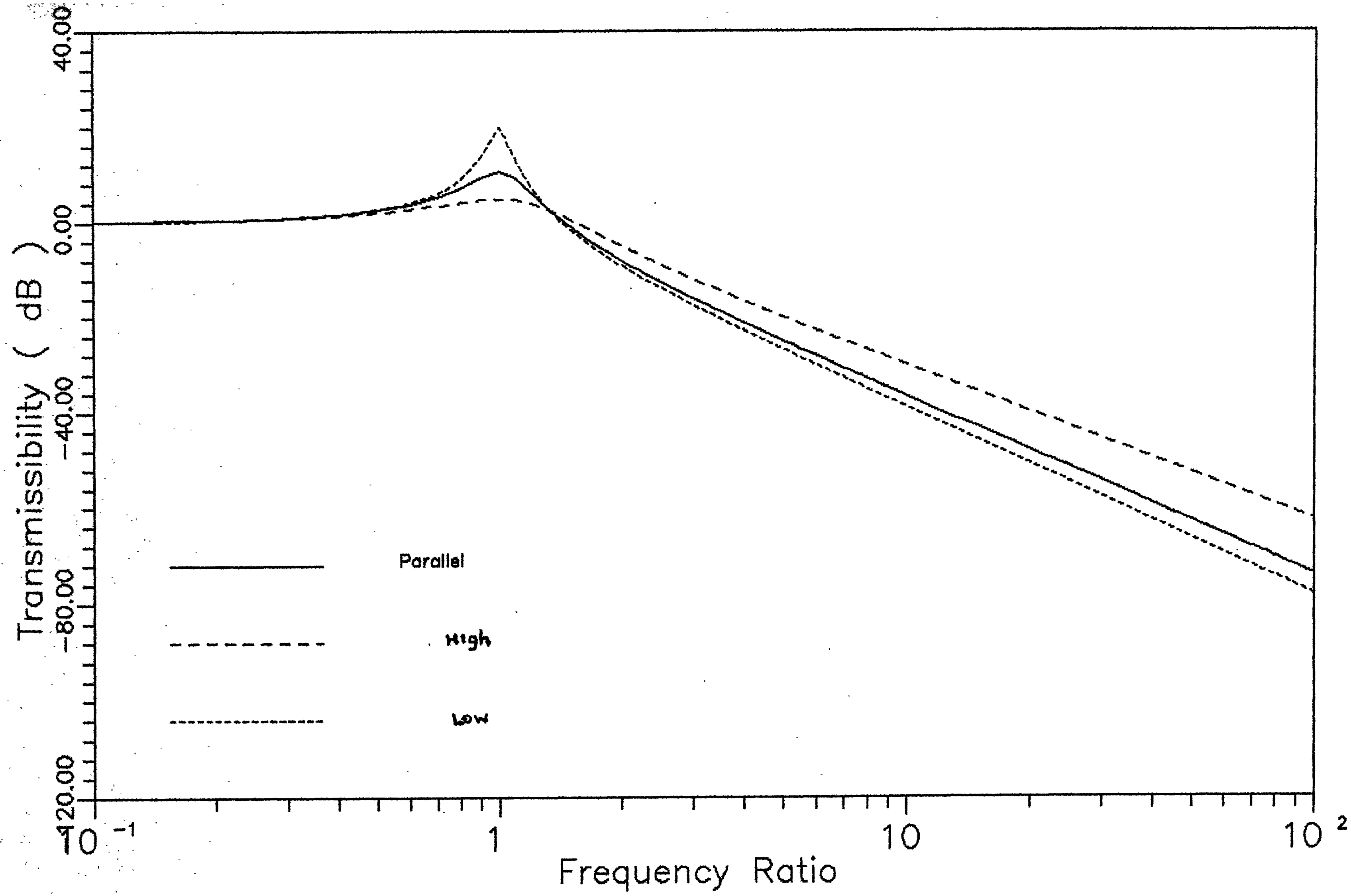

40.00
0.00
-40.00
-80.00
-120.00
Transmissibility (dB)
Parallel
High
Low
10^{-1}
1
10
10^{2}
Frequency Ratio

Fig 3.2-1

Because the dynamic modulus of the high-damping rubber increases rapidly with frequency, its stiffness becomes comparable to that of the low damping rubber and exceeds it only at sufficiently high frequencies. At such frequencies the high frequency attenuation rate of T (expressed normally by dB/octave) decreases. Ultimately the transmissibility decreases at a rate determined by the frequency-dependence of the stiffness of the high damping rubber. However, at such high frequencies transmissibility of the mounting is already significantly less than the transmissibility of the high damping rubber alone. It may be pointed out that, as compared to natural rubber a parallel mounting is more sensitive to changes in the working temperature. Hence a parallel mount is employed most satisfactorily when the working temperature remains fairly constant.

3.3 Foundations of Finite Impedance

3.3.1 General expressions for response ratio of simple mounting system

For the system with a finite impedance foundation, shown in Fig. 3.1-3, the equation of motion for M and the expression for the restoring force exerted by the rubber mounting may be written as

$$M\left(\frac{d^2\tilde{x}}{dt^2}\right) = \tilde{F} - KG^*_\omega(\tilde{x} - \tilde{x}_z) \qquad (3.3.1\text{-}1)$$

and

$$K\,G^*_\omega\,(\tilde{x} - \tilde{x}_z) = \tilde{v}_z/\beta = \left(\frac{j\,\omega\,\tilde{x}_z}{\beta}\right) \qquad (3.3.1\text{-}2)$$

where $\tilde{V}_Z$ and β are respectively, the velocity and point mobility of the foundation. From equs. 3.3.1-1 and 3.3.1-2

$$-\omega^2 M \tilde{x} = \tilde{F} - K G^*_\omega (\tilde{x} - \tilde{x}_Z) \qquad (3.3.1\text{-}3)$$

and $$\tilde{x} = \left(\frac{KG^*_\omega + j\omega/\beta}{KG^*_\omega}\right) \tilde{x}_Z \qquad (3.3.1\text{-}4)$$

eliminating $\tilde{x}$ from equs. 3.3.1-3 and 3.3.1-4

$$\tilde{x}_Z = \frac{KG^*_\omega \tilde{F}}{[KG^*_\omega (j\omega/\beta - \omega^2 M) - \omega^2 M (j\omega/\beta)]} \qquad (3.3.1\text{-}5)$$

if $\tilde{x}_{ZR}$ is the value taken by $\tilde{x}_Z$ when M is connected rigidly to the foundation as shown in Fig. 3.1-4, i.e., the value taken by $\tilde{x}_Z$ when G^*_ω is very large

$$\tilde{x}_{ZR} = \frac{\tilde{F}}{(j\omega/\beta - \omega^2 M)} \qquad (3.3.1\text{-}6)$$

Then,

$$\frac{\tilde{F}_Z}{\tilde{F}_{ZR}} = \frac{\tilde{x}_Z}{\tilde{x}_{ZR}} = \frac{KG^*_\omega}{\{KG^*_\omega - \omega^2 M \left[\frac{1}{(1+j\ \omega M\ \beta)}\right]\}} \qquad (3.3.1\text{-}7)$$

A straight line approximation is often made to the modulus of the measured foundation mobility spectrum, $|\beta|$, plotted on a log-log scale. Then, the modulus of the foundation mobility may be represented to a good approximation, by a law of the form

$$|\beta| = A\ \omega^s \qquad (3.3.1\text{-}8)$$

where A and s are constants with A as positive. The value of s may be estimated from the slope of the log-log plot of the mobility spectrum. A complicated mobility spectrum may be represented adequately by a number of lines, each of the form of eqn. (3.3.1-8). It has been shown in reference [39] that the phase of the mobility spectrum of the above form is given by

$$\emptyset(\omega) = s\pi/2 \qquad (3.3.1\text{-}9)$$

It has been further shown that the value of s must lie in the range $-1 < s < 1$ and the conditions where this holds good are also given. Where $s = \pm 1$, the mobility spectrum corresponds to a stiffness or massline which represents the extreme cases between which all point mobilities of this form must lie.

Therefore, β can be finally expressed as

$$\beta = A\, e^{js\pi/2}\ \omega^{s} = A\omega^{s}[\mathrm{Cos}(s\pi/2 + j\ \mathrm{Sin}(s\pi/2)] \qquad (3.3.1\text{-}10)$$

For example for a beam like foundation $s = -0.5$ and for a plate like foundation $s = 0$. Using eqns. (3.2-1), (3.2-8) and (3.3.1-10) in eqn. (3.3.1-7), Response ratio is obtained finally as

$$R = \left| \frac{\tilde{x}_Z}{\tilde{x}_{ZR}} \right| = \left| \frac{1}{\{1-(\frac{\omega}{\omega_o})^{2-b_1}\left[\frac{1}{(1+j\ \delta G_\omega)} \quad \frac{1}{\{1+j(\frac{\omega}{\omega_o})^{s+1}(\gamma+j\delta)\}}\right.} \right| \tag{3.3.1-11}$$

where γ, δ are the real and imaginary parts of the normalized foundation mobility and are given by

$$\gamma = M\ \omega_o^{s+1}\ A\ \mathrm{Cos}\ \ s\pi/2 \tag{3.3.1-12a}$$

and $$\delta = M\ \omega_o^{s+1}\ A\ \mathrm{Sin}\ s\pi/2 \tag{3.3.1-12b}$$

Larger values of $\sqrt{\gamma^2+\delta^2}$ indicate a very mobile foundation.

The response ratio is plotted against frequency ratio (ω/ω_o), on a log-log scale for beam-like ($s = -0.5$) and plate like ($s = 0$) foundations for different values of γ in Fig. 3.3.1-1 and Fig. 3.3.1-2, respectively.

Both for beam-like and plate-like foundations, it can be seen that the effect of increasing foundation mobility (i.e. increasing γ) has in essence the effect of adding damping to the system. In other words, the peak value at resonance decreases while the high-frequency response ratio increases. It can be further seen that for the plate-like foundation, the high-frequency response ratio is higher than that with beam-like foundation. This is only to be expected since

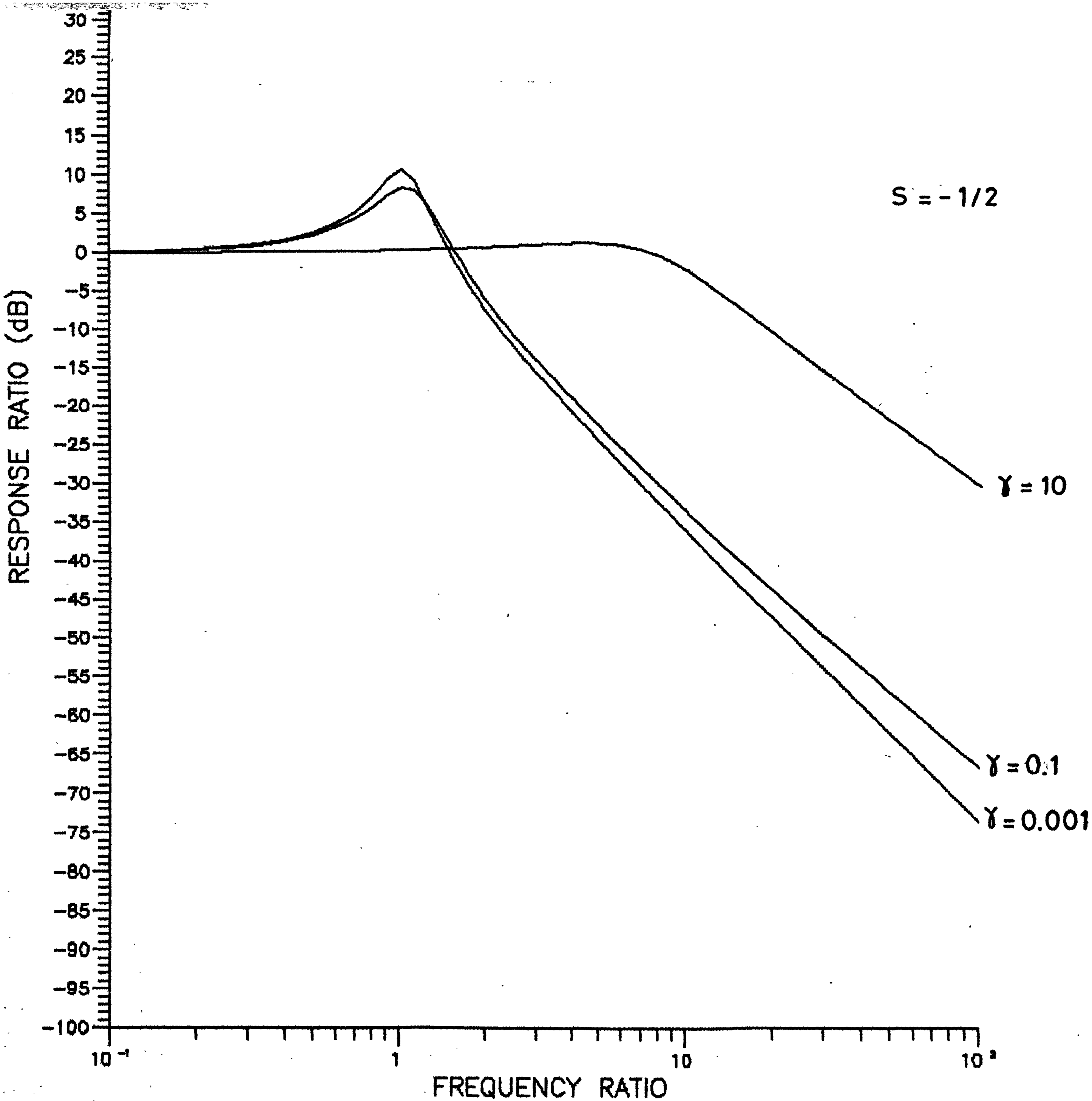

S = -1/2
γ = 10
γ = 0.1
γ = 0.001
RESPONSE RATIO (dB)
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
-60
-65
-70
-75
-80
-85
-90
-95
-100
10⁻¹
1
10
10²
FREQUENCY RATIO

Fig. 3.3.1-1

s=0 implies pure viscous damper like behaviour whereas s= -1/2 implies a combination of a mass and a viscous damper.

If the foundation is very mobile then the resonance peak is absent and the response ratio curve tends to become flat. Therefore, it can be concluded that the effectiveness of an isolator at high frequencies, decreases with increase in the foundation mobility.

3.3.2 General expressions for power flow for simple mounting system

The power flowing into a structure due to harmonic force is dependent on the real part of the mobility of the foundation and is given by

$$P = \frac{1}{2} \; Re\{\beta\}|\tilde{F}_Z|^2 = \frac{1}{2} \; A\omega^s \; |\tilde{F}_Z|^2 \; Cos \; s\pi/2 \qquad (3.3.2\text{-}1)$$

This power flowing into the foundation for the system shown in Fig. 3.1-3 is given by

$$P_Z = \frac{1}{2} \; Re\{\beta\} \; T^2 \; |\tilde{F}|^2 \qquad (3.3.2\text{-}2)$$

where T is the transmissibility across the system. Equation (3.3.2-2) can be written as

$$P_Z = Q_f(\omega) \quad |\tilde{F}|^2 \qquad (3.3.2\text{-}3)$$

where $Q_f(\omega)$ is purely real and depends only on the properties of the mass, isolator and foundation. Substituting expressions

for T and β in eqn. (3.3.2-2)

$$Q_f(\omega) = \frac{\gamma(\frac{\omega}{\omega_o})^s}{2M\,\omega_o} \; \frac{1}{\{|1 - (\frac{\omega}{\omega_o})^{2-b_1} \frac{1}{(1+j\,\delta_{G_\omega})} + j(\frac{\omega}{\omega_o})^{s+1}(\gamma+j\delta)|} \qquad (3.3.2\text{-}4)$$

$Q_f(\omega)$ is plotted on log-log scale with frequency ratio on x-axis for beam like (s = -0.5) and plate like (s=0) foundations for different values of γ and are shown in Fig.3.3.2-1 and Fig. 3.3.2-2, respectively.

Upto certain values of γ, the power flow increases with increase in foundation mobility (i.e. increasing γ) and the power flow curve shifts to higher levels at all frequencies. It can be said that the shape of the curve is primarily governed by the mass-isolator system and whereas the level of the curve is determined by the foundation mobility so long as the foundation is not extremely mobile. This is in agreement with the observation made in Ref. [39]. However, for very mobile foundations, even the shape of the power flow curve is seen to be affected by the foundation mobility as is apparent by the absence of the resonance peak. For very high values of γ, in certain frequency range a flexible foundation may be subjected to less flow of power into it as compared to a more rigid foundation.

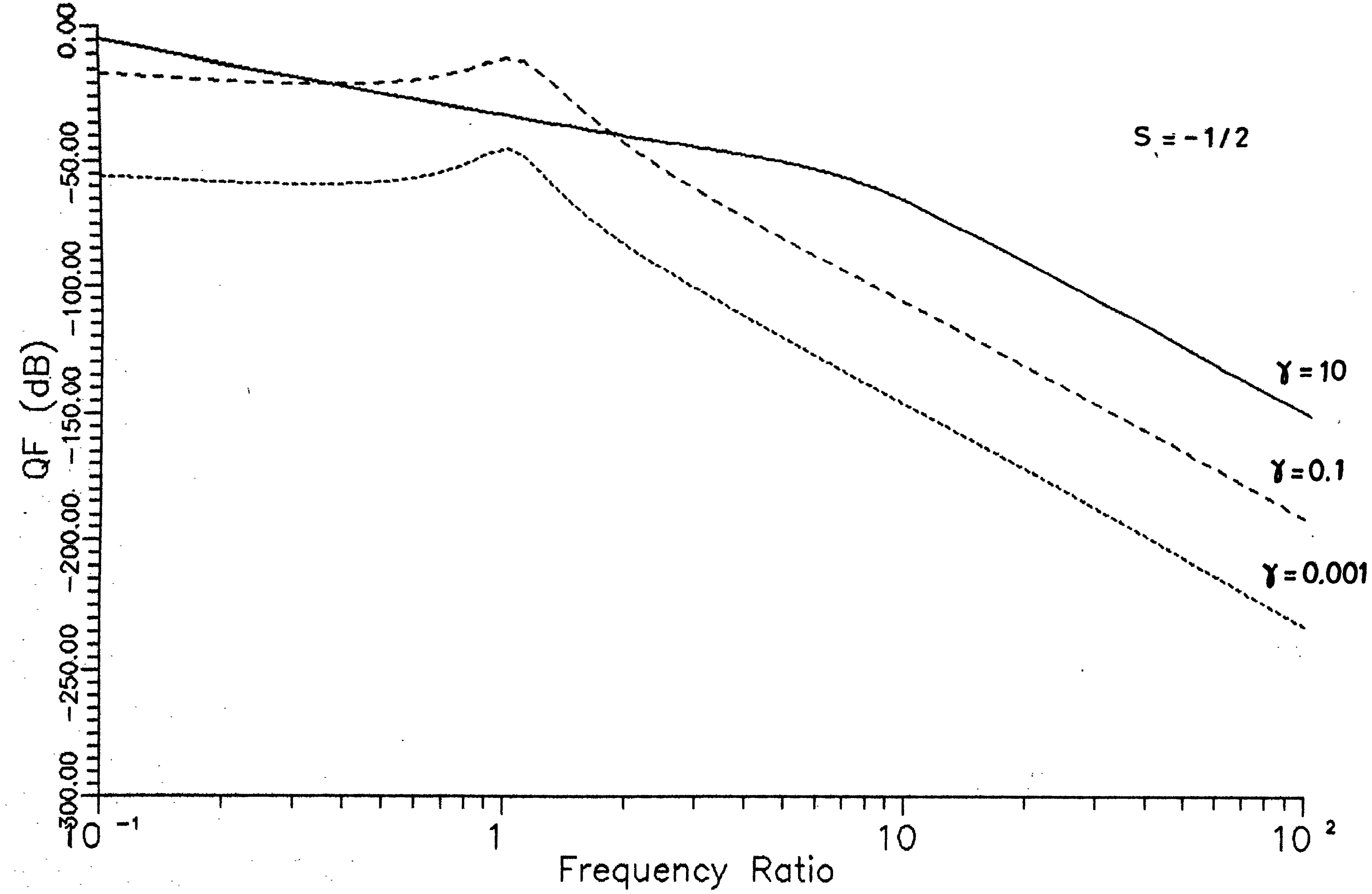

S = -1/2
γ = 10
γ = 0.1
γ = 0.001
QF (dB)
0.00
-50.00
-100.00
-150.00
-200.00
-250.00
-300.00
Frequency Ratio
10 -1
1
10
10 2

Fig. 3.3.2-1

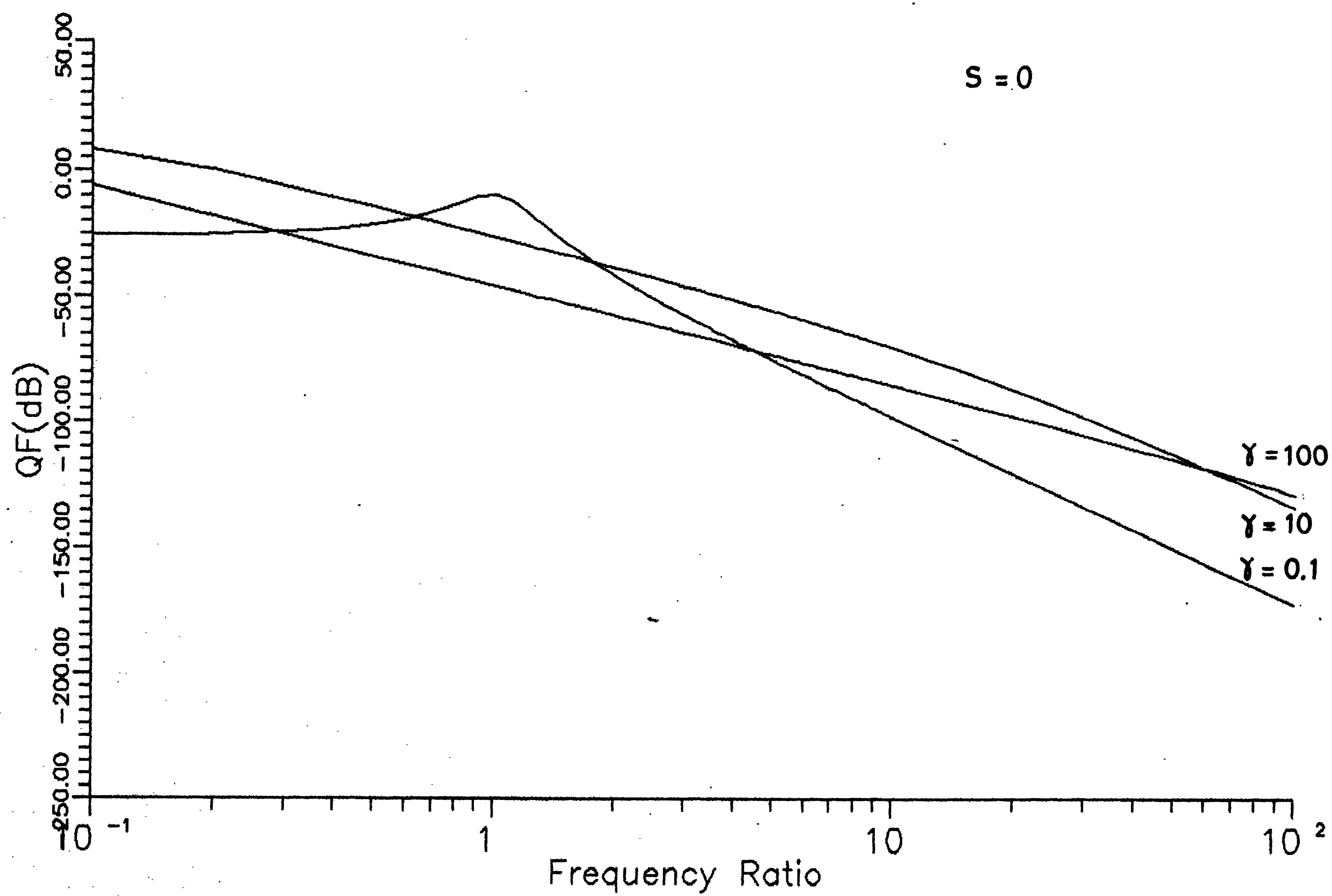

S = 0
γ = 100
γ = 10
γ = 0.1
QF(dB)
50.00
0.00
-50.00
-100.00
-150.00
-200.00
-250.00
10^{-1}
1
10
10^{2}
Frequency Ratio

Fig. 3.3.2-2

It may be mentioned that the power flow is a pertinent index for the design of the foundation but a relevant measure of the effectiveness of the vibration isolator would be the power flow ratio.

For the simple system considered in Fig. 3.1-4 when the mass is rigidly connected to the foundation, the expression for the power flow can be written as

$$P_{ZR} = \frac{1}{2} \operatorname{Re}\{\beta\} \quad |\tilde{F}_{ZR}|^2 \qquad (3.3.2\text{-}5)$$

From eqn. (3.3.2-1) and (3.3.2-1) the expression for power flow ratio, defined in the previous chapter can be obtained as,

$$PF = \frac{P_Z}{P_{ZR}} = \frac{|F_Z|^2}{|F_{ZR}|^2} = \left|\frac{F_Z}{F_{ZR}}\right|^2 = (R)^2 \qquad (3.3.2\text{-}6)$$

Thus the power flow ratio in this case is nothing but the square of the response ratio for the same system. When plotted on a log-log scale the power flow ratio is twice that of the response ratio.

3.4 Wave Effects in Isolators

The simple model described in earlier section is not suitable at high frequency of excitation as the inertia of the isolator cannot be ignored. Wave effects occur at frequencies well above the natural frequency ω_0, and are observed when the mount dimensions become comparable with multiples of the half wavelength of the elastic waves passing through the mounting

It is difficult to take into account, the complicated geometry of rubber isolators. An approximation can be obtained by analysing the mountings that obey the simple wave equation for the longitudinal vibration of a rod of uniform section. Derivation for the transmissibility expressions derived from 'long rod theory' mentioned above can be found in reference [1].

For a rigid foundation, the transmissibility expression so derived is given by

$$T = \left| \frac{1}{\mathrm{Cos}\, n^*\ell - \gamma_2 (n^*\ell) \mathrm{Sin}(n^*\ell)} \right| \tag{3.4-1}$$

where $n^*\ell = p + jq$, $\gamma_2 = M/M_R$

$$p = \frac{n\ell}{(1+\delta E_\omega^2)^{1/2}} \left(\frac{E_o}{E_\omega}\right)^{1/2} \left[\frac{(1+\delta E_\omega^2)^{1/2} + 1}{2} \right]^{1/2}$$

$$q = \frac{-n\ell}{(1+\delta E_\omega^2)^{1/2}} \left(\frac{E_o}{E_\omega}\right)^{1/2} \left[\frac{(1+\delta E_\omega^2)^{1/2} - 1}{2} \right]^{1/2}$$

$$n = \omega(\rho / E_o)^{1/2}$$

where M is the mass of the machine,

M_R is the mass of the isolator $= \rho a \ell$

a is the cross-sectional area of the isolator

ℓ is the length of rod like isolator

ρ is the density of the mount material

E_o is the value taken by dynamic Young's modulus E_ω, when $\omega = \omega_o$

δE_ω is the damping factor.

$(n\ell)$ can be expressed as function of (ω/ω_o) as

$$n\ell = \left(\frac{\omega}{\omega_o}\right)\psi$$

where $\psi = (M_R/M)^{1/2}$.

If E_ω is assumed to be of the form $a_1\ \omega^{b_1}$ and δE_ω is constant and substituting the expressions for $(n\ell)$, expressions for p and q are modified as given below.

Defining $D=(1+\ \delta E_\omega^2)^{1/2}$ and $\gamma_2 =\ M/M_R$, one can write

$$p = \left(\frac{\omega}{\omega_o}\right)^{1-b_1/2} \frac{1}{D}\ \frac{1}{\sqrt{\gamma_2}}\ \left[\frac{D+1}{2}\right]^{1/2}$$

$$q = -\left(\frac{\omega}{\omega_o}\right)^{1-b_1/2} \frac{1}{D}\ \frac{1}{\sqrt{\gamma_2}}\ \left[\frac{D-1}{2}\right]^{1/2}$$

This result is only used to provide a check for the analysis of a more general system, presented in the next section, a limiting situation of which reduces to the system discussed above.

3.5 Systems With Periodicity

In this section, vibration isolation of systems which have some kind of periodicity are discussed. The concept of periodicity, as used in the studies on vibration of periodic structures, is used. By periodicity, it is meant that either the entire system or the isolator consists of a number of identical elements joined end to end. Two different situations of practical significance are included. These are shown in Figs. 3.5-1 and 3.5.-2.

The usual practice in the design of a single degree-of-freedom isolator is to augment the mass of the machine by bolting it to a concrete bed, the mass of which is several times that of the machine itself, and then mount the assembly on an isolator. The total mass can be subdivided into several equal masses, separated by isolator elements as shown in Fig. 3.5.1b and c. This results in a periodic vibration isolation system. Response ratio of this system is calculated for not a very mobile foundation in section 3.5.-2.

In section 3.4, the isolator is modelled as a rodlike-mount, but in practice the geometry of isolator is quite complex and contains intermediate steel spacers. These isolators are assumed to be periodic and are referred to as periodic isolators. A machine supported on a periodic isolator is shown in Fig. 3.5.-2 The effect of the steel spacers on wave effects in these periodic isolators is examined in section 3.5-3.

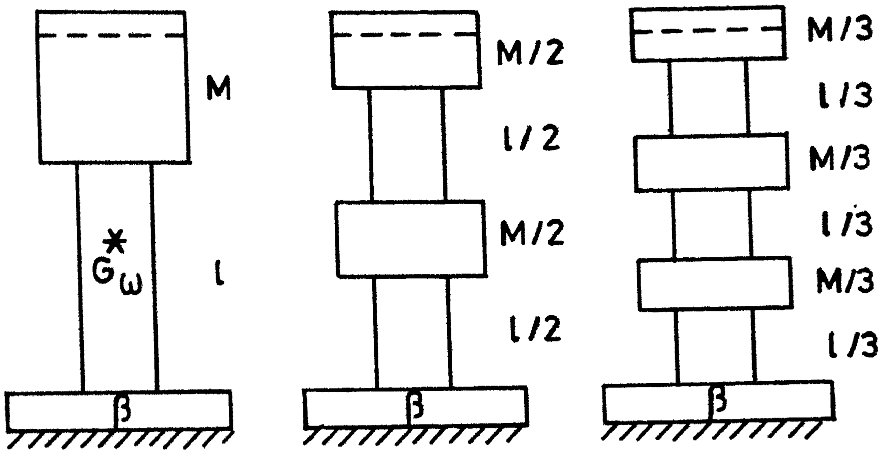


Fig. 3.5-1

Periodic Isolation System

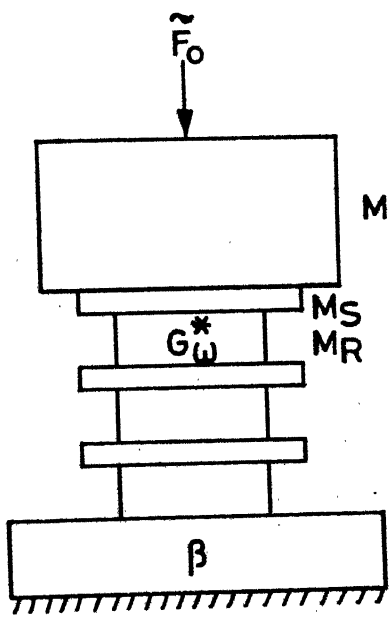


Fig. 3.5-2

Periodic Isolator

3.5.1 Mobility analysis of a periodic isolation system on an infinite-impedance foundation

In Fig. 3.5.1-1, an 'n element' periodic isolation system acted upon by a harmonic force is shown. Considering three successive elements one can write

$$V_m = \beta_{r\ell} F_{m-1} + \beta_{rr} F_m \qquad (3.5.1\text{-}1a)$$

and

$$V_m = \beta_{\ell r} F_{m+1} + \beta_{\ell\ell} F_m \qquad (3.5.1\text{-}1b)$$

where V_m, F_{m-1}, F_m and F_{m+1} are shown in Fig. 3.5.1-1. $\beta_{r\ell}$, β_{rr}, $\beta_{\ell r}$, $\beta_{\ell\ell}$ are the mobilities whose first subscript denotes the position of velocity and the second subscript denotes that of force. Subscripts ℓ and r refer to the left and right, respectively. Evaluation of these mobilities is detailed in Appendix B.

Elimination of V_m from eqns. (3.5.1-1a and b) results in a difference equation given by

$$\beta_{\ell r} F_{m+1} + (\beta_{\ell\ell} - \beta_{rr}) F_m - \beta_{r\ell} F_{m-1} = 0 \qquad (3.5.1\text{-}2)$$

The solution of this equation can be assumed to be of the form

$$F_m = F_{m-1}\ \mu \qquad (3.5.1\text{-}3)$$

Substituting the above equation in eqn. (3.5.1-2), we get

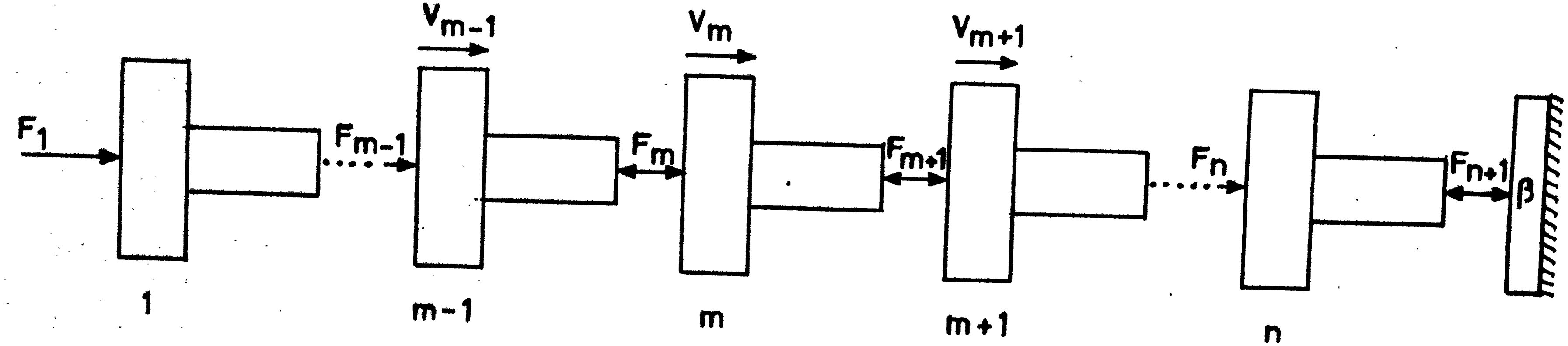


Fig. 3.5.1-1

Periodic Isolation System.

$$\mu^2 \beta_{\ell r} + (\beta_{\ell\ell} - \beta_{rr})\mu - \beta_{r\ell} = 0 \qquad (3.5.1\text{-}4)$$

Equation (3.5.1-4) has two roots μ_1 and μ_2, given by

$$\mu_{1,2} = \frac{-(\beta_{\ell\ell} - \beta_{rr}) \pm \sqrt{(\beta_{\ell\ell} - \beta_{rr})^2 + 4\,\beta_{r\ell}\,\beta_{\ell r}}}{2\beta_{\ell r}} \qquad (3.5.1\text{-}5)$$

Considering the n element periodic isolation system shown in Fig. 3.5.1-1, one can write from eqn. (3.5.1-3)

$$F^1_{n+1} = F^1_1\ \mu^n_1 \qquad (3.5.1\text{-}6a)$$

and
$$F^2_{n+1} = F^2_2\ \mu^n_2 \qquad (3.5.1\text{-}6b)$$

where F^1_{n+1} and F^2_{n+1} are the solutions corresponding to μ_1 and μ_2, respectively.

From eqns. (3.5.1-1) and (3.5.1-6) it can be written that

$$V^1_{n+1} = \beta_{rr} F^1_{n+1} + \beta_{r\ell} F^1_n = F^1_{n+1} (\beta_{rr} + \beta_{r\ell}/\mu_1) \qquad (3.5.1\text{-}7a)$$

and similarly
$$V^2_{n+1} = F^2_{n+1} (\beta_{rr} + \beta_{r\ell}/ \mu_2) \qquad (3.5.1\text{-}7b)$$

Since the general solution of a difference equation is a linear combination of all the possible solutions one gets

$$F_1 = F^1_1 + F^2_1 \qquad (3.5.1\text{-}8a)$$

and
$$F_{n+1} = F^1_{n+1} + F^2_{n+1} \qquad (3.5.1\text{-}8b)$$

Now imposing the end condition for a rigid foundation i.e., putting $V_{n+1} = 0$ one obtains

$$V^1_{n+1} + V^2_{n+1} = 0 \qquad (3.5.1\text{-}9)$$

Substituting for V^1_{n+1} and V^2_{n+1} from eqns. (3.5.1-7a) and (b) it can be shown from eqn. (3.5.1-9) that

$$\frac{F^1_{n+1}}{F^2_{n+1}} = -\frac{(\beta_{rr} + \beta_{r\ell}/\mu_2)}{(\beta_{rr} + \beta_{r\ell}/\mu_1)} = K' \text{ (say)} \qquad (3.5.1\text{-}10)$$

Finally, transmissibility T is obtained as

$$T = \left|\frac{F_{n+1}}{F_1}\right| = \left|\frac{F^1_{n+1} + F^2_{n+1}}{F^1_1 + F^2_1}\right| \qquad (3.5.1\text{-}11)$$

Using eqns. (3.5.1-10) and (3.5.1-6a) and (b) in the above equation

$$T = \left|\frac{1 + K'}{\mu_2^{-n} + K' \mu_1^{-n}}\right| \qquad (3.5.1\text{-}12)$$

The above equation can be shown to reduce to eqn. 3.4.1, as it should when the periodic isolation system has one element $(n = 1)$. For $n > 1$, eqn. (3.5.1-11) is compared with the transmissibility expressions given in reference [68] and it is found that they are in agreement with each other. However, it should be mentioned that transmissibility expressions derived in the above reference are quite cumbersome for increasing

values of n and the coefficients of a polynomial expression have to be evaluated for each value of n separately. Equation (3.5.1-11) is quite simple and the computational effort is independent of number of elements. This is significant with increasing values of n.

3.5.2 Mobility analysis of a periodic isolation system for a finite impedance foundation

When the foundation of Fig. 3.5-1 is of finite impedance, the end condition given by eqn. (3.5.1-9) for a rigid foundation should be replaced by

$$F_{n+1} = \frac{V_{n+1}}{\beta} \tag{3.5.2-1}$$

where β is the foundation mobility.

From eqns. (3.5.1-7), (3.5.1-8), and (3.5.2-10) we get ratio K' as

$$K' = \frac{F^1_{n+1}}{F^2_{n+1}} = \frac{-[\beta - (\beta_{rr} + \beta_{r\ell}/\mu_2)]}{[\beta - (\beta_{rr} + \beta_{r\ell}/\mu_1)]} \tag{3.5.2-2}$$

Substituting the expression for β and carrying out the non-dimensionalisation, one gets finally

$$K' = -\left[\frac{(\omega/\omega_o)^s(\gamma + j\delta) - (\beta_{rr} + \beta_{r\ell}/\mu_2)\gamma_2 M_R\, \omega_o}{(\omega/\omega_o)^s (\gamma + j\delta) - (\beta_{rr} + \beta_{r\ell}/\mu_1)\gamma_2 M_R\, \omega_o}\right] \tag{3.5.2-3}$$

where

$$\gamma_2 = \frac{M}{M_R} \quad \text{with } M_R = \rho a \ell$$

The expression for the transmissibility T is still given by eqn. (3.5.1-12).

Expressions for response ratio and power flow can be easily obtained from eqns. (3.3.1-11) and (3.3.2-3) as

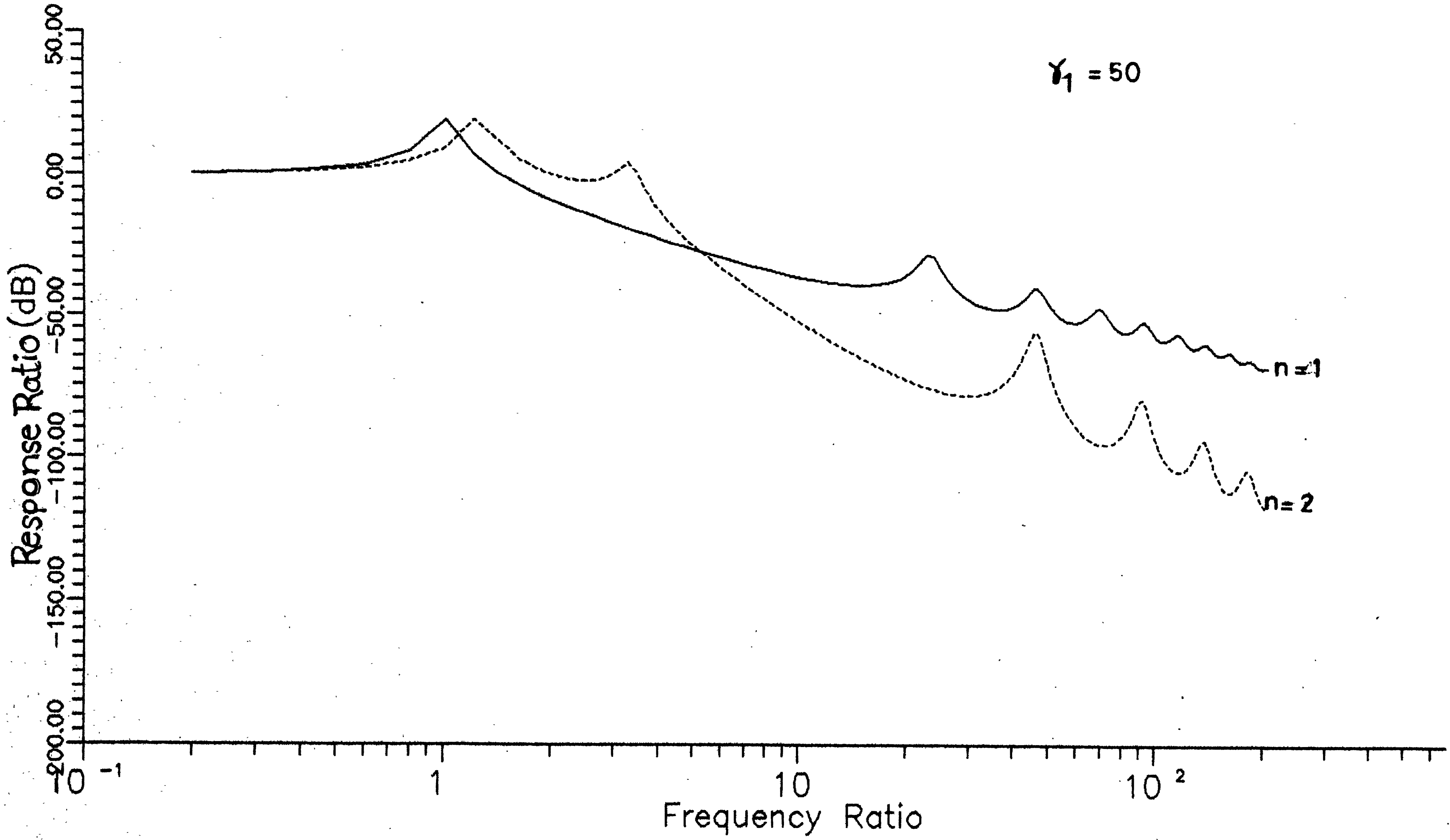
$$R = \left|\frac{\tilde{F}_{n+1}}{\tilde{F}_{ZR}}\right| = (T) \left| 1 + j\ (\omega/\omega_o)^{s+1}\ n\ (\gamma + j\delta) \right| \qquad (3.5.2\text{-}4)$$

and

$$P = \frac{\gamma\,(\omega/\omega_o)^s}{2M\ \omega_o}\ (T)^2\ |F|^2 \qquad (3.5.2\text{-}5)$$

respectively.

The response ratio is evaluated for three cases shown in Figs. 3.5.1, when γ_2, the mass ratio of the machine and isolator is 50 and the isolator material is a low damping natural rubber whose properties are given by $G_\omega = 15x10^5$ N/m^2 and $\delta G_\omega = 0.1$ and for not a very mobile foundation. Results are plotted in Figs. 3.5.2-1 and Fig. 3.5.2-2. It can be seen that as the number of periodic elements increases, the high frequency attenuation rate of response ratio increases. This is in agreement with reference [64] where the damping and inertia of the isolator were ignored. The present analysis, however, due to the inclusion of the wave effects shows the



$\gamma_1 = 50$
n=1
n=2
Response Ratio (dB)
50.00
0.00
−50.00
−100.00
−150.00
−200.00
10^{-1}
1
10
10^{2}
Frequency Ratio

Fig. 3.5.2-1

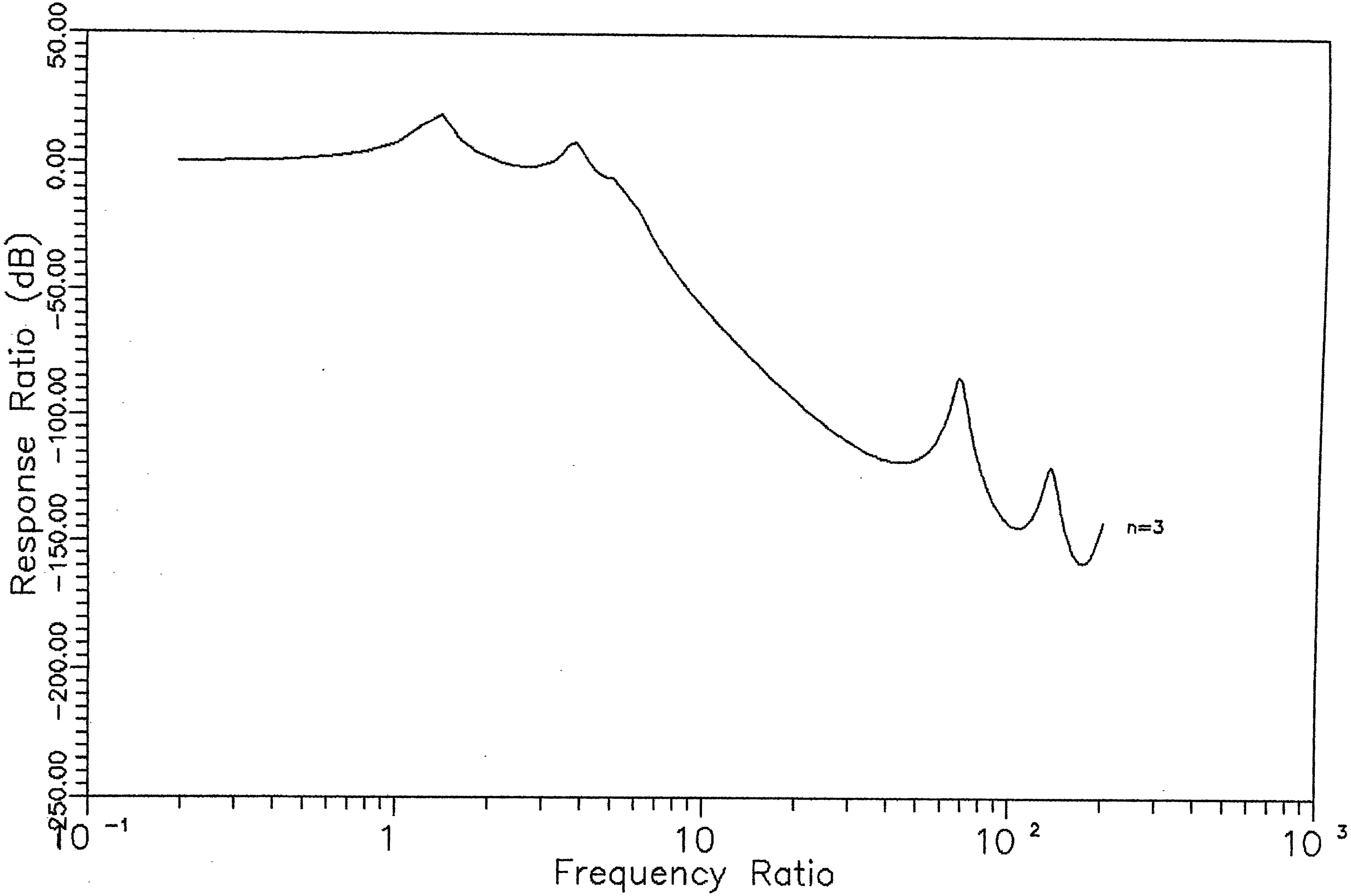

Response Ratio (dB)
50.00
0.00
-50.00
-100.00
-150.00
-200.00
-250.00
Frequency Ratio
10 -1
1
10
10 2
10 3
n=3

Fig 3.5.2-2

presence of new resonance peaks with increasing number of periodic elements. Though the envelope, of the response ratio still drops at a faster rate with increase in 'n', the peaks at wave resonances of the isolator become sharp and may even nullify the advantage of the system at high frequencies. However, with a parallel rubber mount with effectively high damping, these wave resonances get smoothened out and a periodic isolation system performs better than a simple one element system.

3.5.3 Periodic isolators

A machine mounted on a periodic isolator is shown in Fig. 3.5.2. In this figure,

M refers to the mass of the machine,

M_S refers to the mass of the steel spacer, and

M_R refers to the mass of the rubber block.

The governing equation of motion for the machine is

$$j\omega M \tilde{V}_1 = F_o - \tilde{F}_1 \tag{3.5.3-1}$$

Using eqns. (3.5.1-3), (3.5.1-6), (3.5.1-7), (3.5.1-8) and (3.5.2-10), and substituting $\tilde{V}_1$ and $\tilde{F}_1$ in terms of F_{n+1}, eqn. (3.5.3-1) becomes

$$F^1_{n+1}\,[B_1] + F^2_{n+1}\,[B_2] = F_o \tag{3.5.3-2}$$

where B_1, B_2 are defined as follows:

$$B_1 = j\omega M\,(\beta_{\ell r} + \beta_{\ell\ell}/\mu_1)\ \frac{1}{\mu_1^{n-1}} + \frac{1}{\mu_1^{n}} \qquad (3.5.3\text{-}3a)$$

and $$B_2 = j\omega M\,(\beta_{\ell r} + \beta_{\ell\ell}/\mu_2)\ \frac{1}{\mu_2^{n-1}} + \frac{1}{\mu_2^{n}} \qquad (3.5.3\text{-}3b)$$

Equations (3.5.3-3a) and (b) are nondimensionalised to yield

$$B_1 = j(\omega/\omega_o)\,(\beta_{\ell r} + \beta_{\ell\ell}/\mu_1)\ \frac{1}{\mu_1^{n-1}}\,\gamma_2 M_R \omega_o + \frac{1}{\mu_1^{n}} \qquad (3.5.3\text{-}4a)$$

and

$$B_2 = j(\omega/\omega_o)\,(\beta_{\ell r} + \beta_{\ell\ell}/\mu_2)\ \frac{1}{\mu_2^{n-1}}\,\gamma_2 M_R \omega_o + \frac{1}{\mu_2^{n}} \qquad (3.5.3\text{-}4b)$$

where n = 1, the above equations can be simplified as

$$B_1 = j(\omega/\omega_o)\,(\beta_{\ell r} + \beta_{\ell\ell}/\mu_1)\gamma_2 M_R \omega_o + \frac{1}{\mu_1} \qquad (3.5.3\text{-}5a)$$

and

$$B_2 = j(\omega/\omega_o)\,(\beta_{\ell r} + \beta_{\ell\ell}/\mu_2)\gamma_2 M_R \omega_o + \frac{1}{\mu_2} \qquad (3.5.3\text{-}5b)$$

For an n element isolator, using eqns. (3.5.1-10) and (3.5.3-2), the transmissibility T is finally obtained as

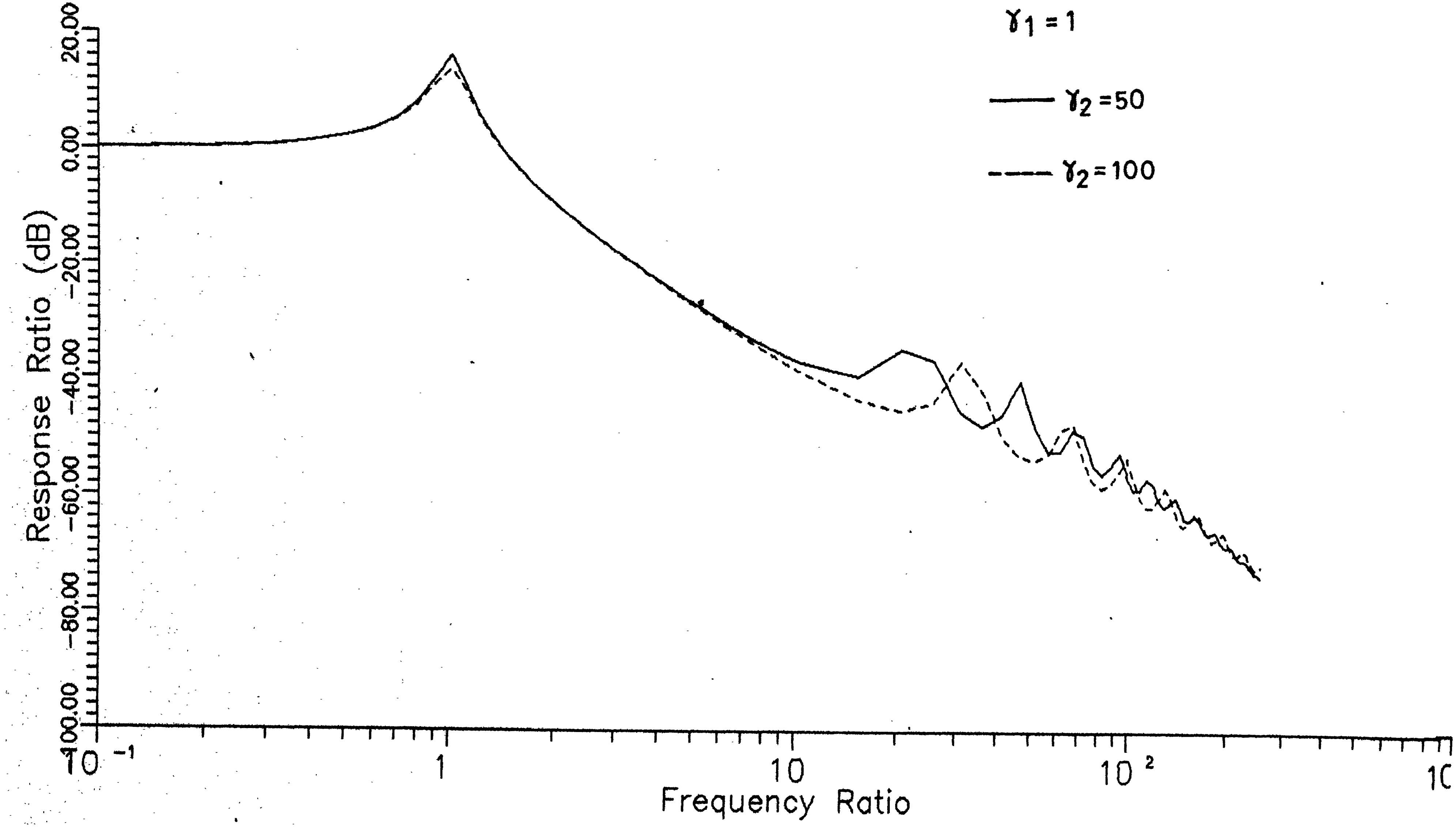
$$T = \left|\frac{F_{n+1}}{F_o}\right| = \left|\frac{1 + K'}{K' B_1 + B_2}\right| \qquad (3.5.3\text{-}6)$$

Expression for response ratio can be obtained as

$$R = (T) \quad | 1 + j(\omega/\omega_o)^{s+1} (\gamma + j\delta) | \qquad (3.5.3\text{-}7)$$

Expression for power flow is still given by the eqn. (3.5.2-5). The response ratio for the case n = 1, are shown in Fig. 3.5.3-1, for a low damping natural rubber with various values $\gamma_2 = M/M_R$ and $\gamma_1 = M_S/M_R$. It can be observed that as the mass ratio of the machine to the isolator, i.e. γ_2 increases the wave resonances shift to higher frequencies as the inertia effects of the isolators become dominant only at shorter wavelengths. Figure 3.5.3-2 shows the response ratio plot for a parallel mounting rubber. It can be seen that the wave resonances are not apparent as the peaks are smoothened out by the relatively high value of the damping factor of parallel mounting.

The response ratio, for the three cases shown in Fig. 3.5.3-3, is shown in Fig. 3.5.3-4 and Fig. 3.5.3-5. It can be seen that the intermediate steel spacer results in a better performance of the isolator when its inertia is significant. Though the high frequency attenuation rate of the response ratio increases with 'n', the peaks at wave resonances become sharp and it may prove to be counter effective. Therefore, for a low damping rubber, for the same material and space of rubber, it may not be advisable to go for more than two elements (n = 2).



$\gamma_1 = 1$
$\gamma_2 = 50$
$\gamma_2 = 100$
Response Ratio (dB)
20.00
0.00
-20.00
-40.00
-60.00
-80.00
-100.00
10^{-1}
1
10
10^{2}
Frequency Ratio

Fig. 3.5.3-1

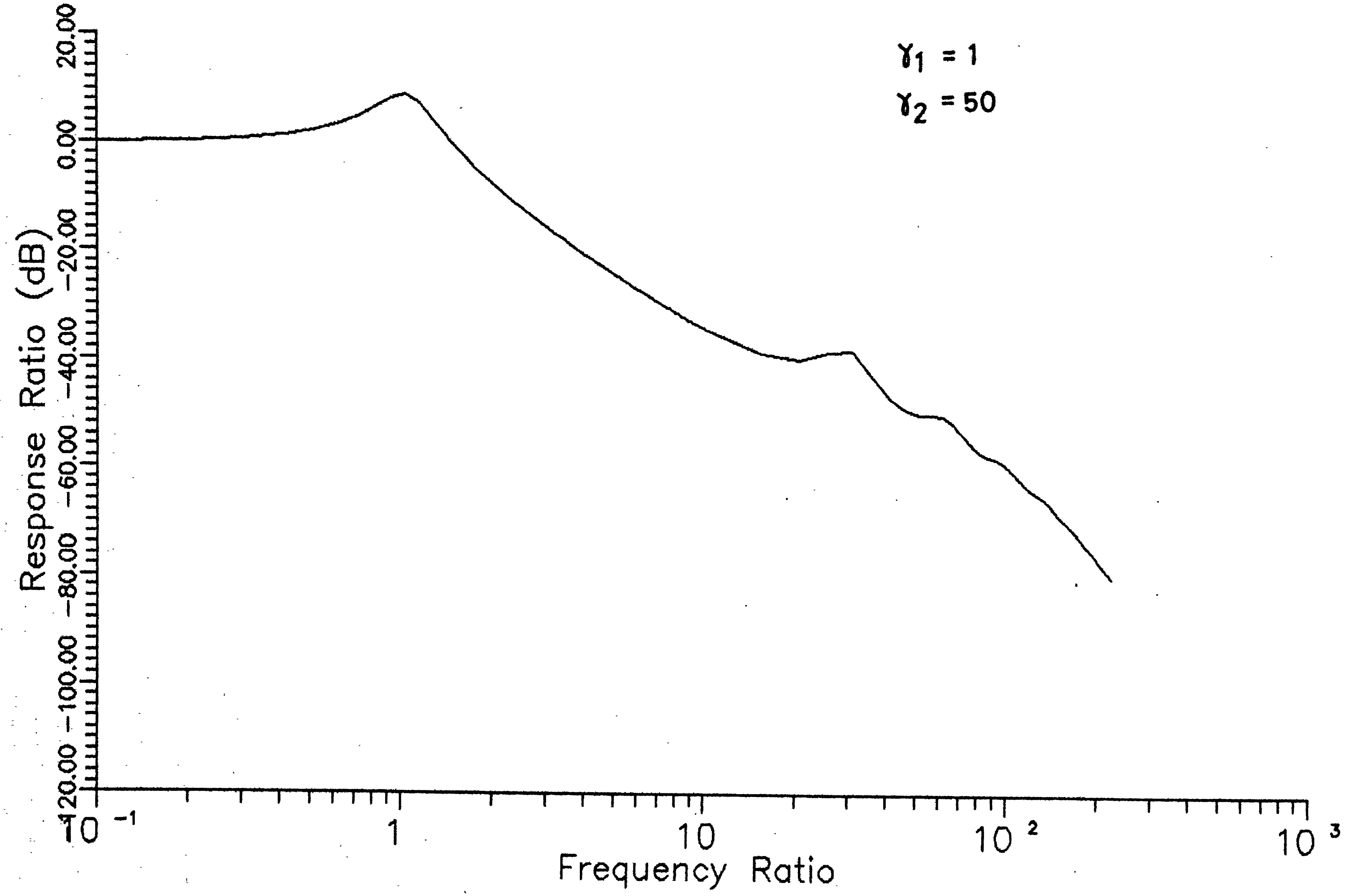

$\gamma_1 = 1$
$\gamma_2 = 50$
20.00
0.00
-20.00
-40.00
-60.00
-80.00
-100.00
-120.00
Response Ratio (dB)
10^{-1}
1
10
10^{2}
10^{3}
Frequency Ratio

Fig. 3.5.3-2

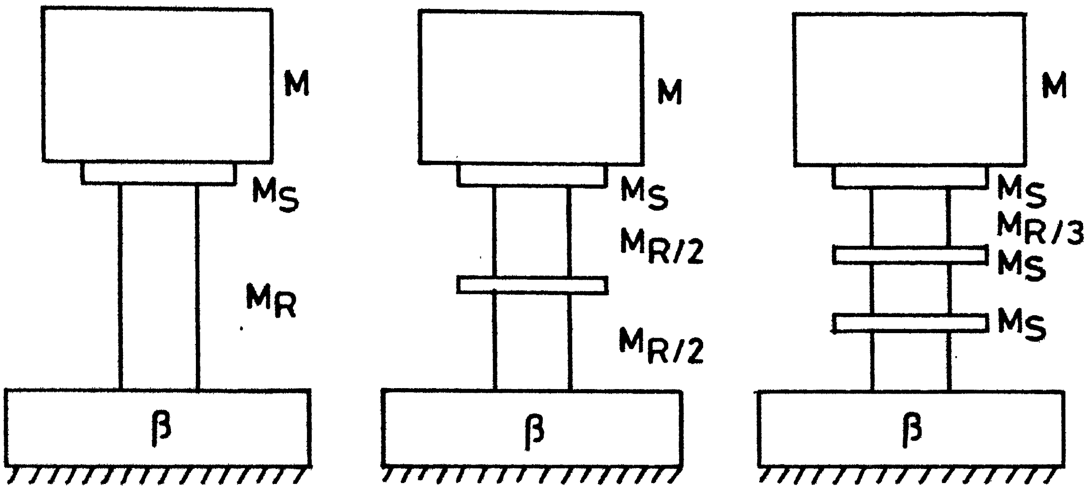


Fig. 3.5.3-3

Periodic Isolators

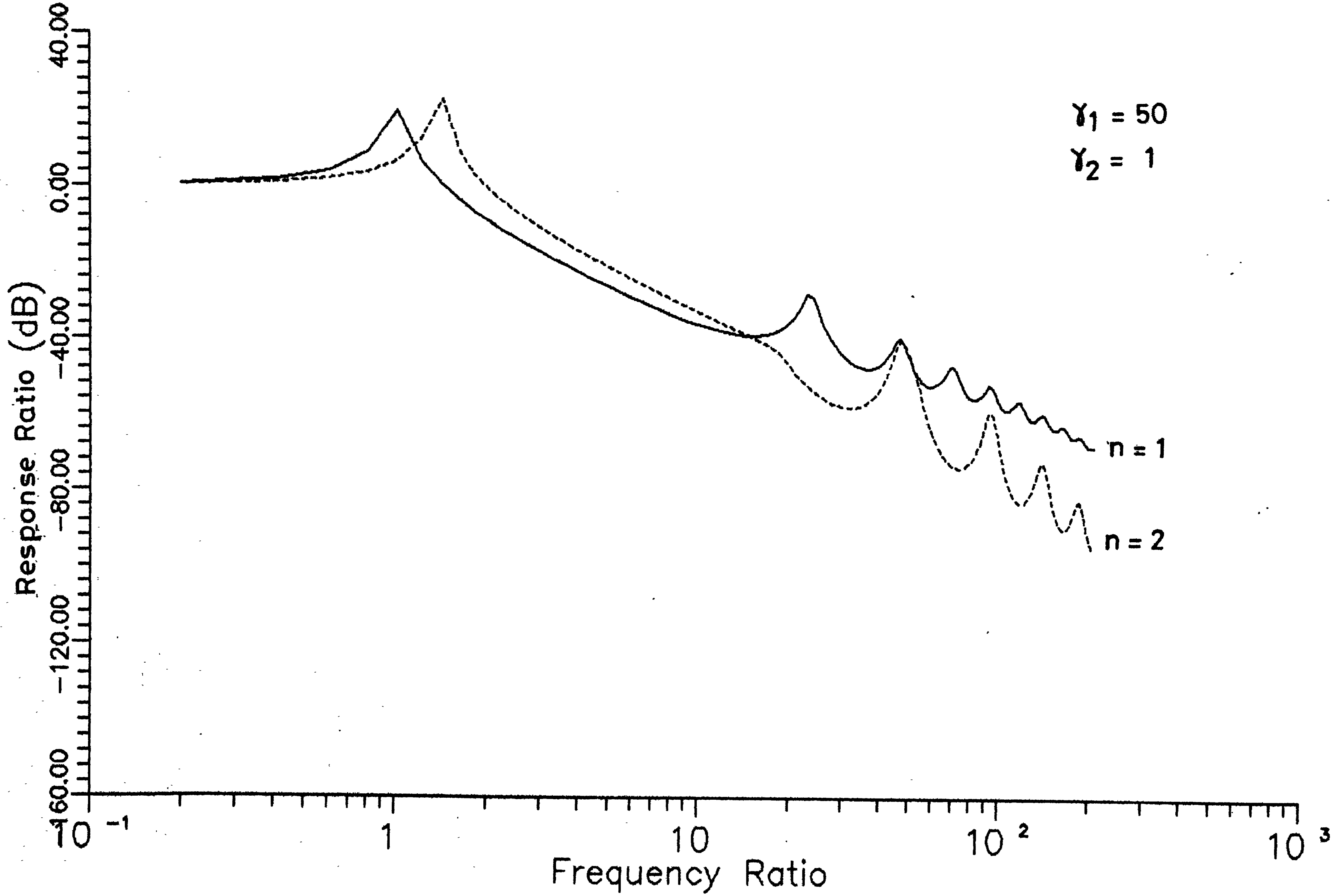

$\gamma_1 = 50$
$\gamma_2 = 1$
n = 1
n = 2
Response Ratio (dB)
40.00
0.00
-40.00
-80.00
-120.00
-160.00
10^{-1}
1
10
10^{2}
10^{3}
Frequency Ratio

Fig. 3.5.3-4

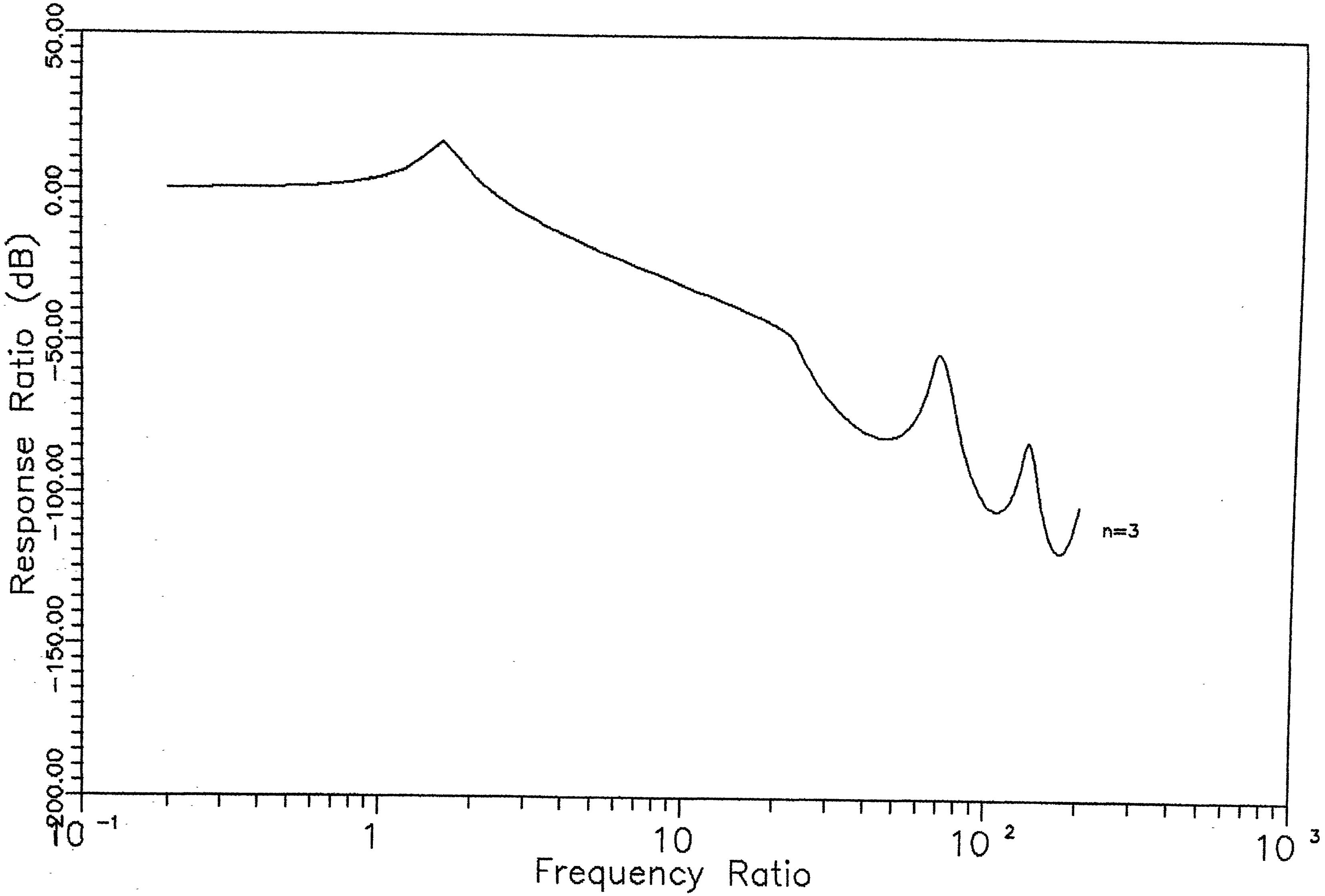

Response Ratio (dB)
50.00
0.00
−50.00
−100.00
−150.00
−200.00
Frequency Ratio
10 −1
1
10
10 2
10 3
n=3

Fig 3.5.3−5

CHAPTER IV

TWO POINT EXCITATION MODEL

4.1 Introduction

In the previous chapter, the principle of vibration isolation for a rigid body which is constrained to move in one direction was discussed. Thus the system had only single degree-of-freedom with one natural frequency. An unconstrained rigid body moving in space, however, has six degrees of freedom. The complete description of vibration of such a body requires six coordinates. Normally, three coordinates are used to describe the translation of a point on the rigid body along three mutually perpendicular axes and the other three coordinates describe the rotation of the rigid body about these three axes. Each of these coordinates refers to a 'mode' of vibration. Thus, in general, an unconstrained rigid body on massless isolators has six degrees of freedom and six natural frequencies. The different modes of vibration of such an isolator-rigid body system may be coupled. The coupling implies that vibration in one of these modes simultaneously generates all the coupled modes. The natural frequency pertaining to an uncoupled mode is referred to as an uncoupled natural frequency. If n number of modes are coupled, then a set of n natural frequencies, called coupled natural frequencies, apply to all such modes. The complicated

analysis of the most general system, where all the six modes are coupled, is of hardly any use in engineering practice [33]. Fortunately, certain symmetry conditions, normally encountered in practice are assumed without significant error, decouple certain modes of vibration and the analysis is greatly simplified, The extent of simplication depends on the degree of symmetry.

In the previous chapter, the isolator stiffness was represented by an ideal spring which deformed only in the direction of the applied force. In general the stiffness of the isolator depends both on the direction (along which it is measured) and the applied constraints. The axes of geometrical symmetry are the principal elastic axes of the isolator.

The transmissibility for a system with many degrees of freedom depends on the nature and location of excitation. Hence no general result can be derived and each problem has to be analysed separately. In the following sections transmissibility characteristics are discussed with reference to systems having two planes of symmetry. The orientation of principal elastic axes of the isolators are taken to be parallel to those of the principal axes of moments of inertia of the rigid body. The effects of frequency dependent stiffness and damping of the isolator material, isolator inertia and finite impedance of the foundation on transmissibility are also discussed.

4.2 Analysis of Two-Planes of Symmetry Model

Consider the system shown in Fig. 4.2-1a. A rigid body is supported by four identical isolators at its bottom corners. The entire system has two vertical planes of symmetry, XZ and YZ, passing through the center of mass, 0, of the rigid body. The principal axes of moments of inertia of the rigid body are the coordinate axes X, Y and Z. These three coordinate axes are also parallel to the principal elastic axes of the isolators. The principal stiffnesses of each isolator is $K_X G_\omega^*$, $K_Y G_\omega^*$ and $K_Z G_\omega^*$. The location of each isolator is described by the coordinates of its mid point a_{X_i}, a_{Y_i} and a_{Z_i} with $i = 1,2,3$. Due to the symmetrical location of the isolators, one can write

$$a_{X_1} = a_{X_2} = - a_{X_3} = - a_{X_4} = a_X \tag{4.2-1a}$$

$$a_{Y_1} = -a_{Y_2} = -a_{Y_3} = a_{Y_4} = a_Y \tag{4.2-1b}$$

$$a_{Z_1} = a_{Z_2} = a_{Z_3} = a_{Z_4} = -a_Z \tag{4.2-1c}$$

where a_X, a_Y, a_Z are positive quantities.

The point mobility of foundation at all the four contact points is assumed to be identical and the transfer mobilities are assumed to be negligible. The foundation is assumed to be ideally rigid in all directions except in the vertical direction.

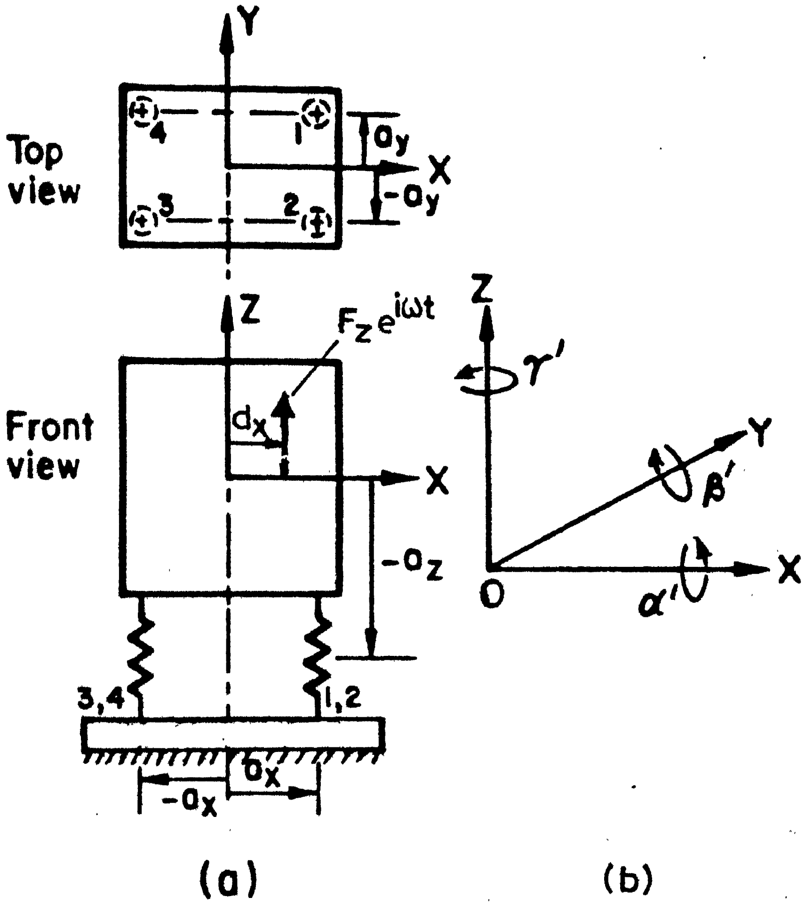


Fig. 4.2-1

Two Point Excitation Model

Referring to Fig. (4.2-1b) let x, y and z refer to the translations of the center of mass O along X, Y and Z axes, respectively. The rotations of the rigid body about X, Y and Z axes are denoted, respectively by α', β' and γ'. All movements are measured from static equilibrium position. For small motion, the deformations of the ith isolator along X, Y, Z directions are then given, respectively by

$$\delta x_i = x + a_{z_i}\beta' - a_{y_i}\gamma' \qquad (4.2\text{-}2a)$$

$$\delta y_i = y + a_{x_i}\gamma' - a_{z_i}\alpha' \qquad (4.2\text{-}2b)$$

$$\delta z_i = z + a_{y_i}\alpha' - a_{x_i}\beta' - z_{b_i} \qquad (4.2\text{-}2b)$$

where z_{b_i} is the foundation displacement at isolator position i.

Before discussing the harmonic force transmissibility of the system shown in Fig. (4.2 -1), one should note certain basic differences in the transmissibility characteristics of this system with those of the single degree of freedom system discussed in the previous chapter. First of all, due to possible coupling of various modes in this multi degrees of freedom system, the force transmitted by an isolator need not necessarily be in the direction of the exciting force.

Secondly, due to rotational motion of the rigid body, all the isolators do not undergo identical deformation. Consequently the transmissibility of each isolator is to be obtained separately. Furthermore, the transmissibility expression also depends on the direction and location of of the exciting force as these decide the modes that are going to be excited.

In the Fig. (4.2-1) a harmonic exciting force $F_z \, e^{j\omega t}$ acts along the z-direction at a point A (d_x, 0, 0), then the rigid body is subjected to a moment about the Y-axis. As a result, the rigid body undergoes rotational motion β'. Due to coupling of β' and x modes, the rigid body is thus excited simultaneously to all three modes, namely z, x, and β'. The equations of motion corresponding to these three modes can be derived by considering the free body diagrams of rigid body and isolators.

From symmetry condition it can be assumed that

$$z_{B_1} = z_{B_2} \quad \text{and} \quad z_{B_3} = z_{B_4} \tag{4.2-3}$$

From equations (4.2-3) and equations (4.2-1), equations (4.2-2) can be written as

$$\delta z_1 = \delta z_2 = z - a_x \, \beta' - z_{B_1} \tag{4.2-4a}$$

$$\delta z_3 = \delta z_4 = z + a_x \, \beta' - z_{B_3} \tag{4.2-4b}$$

$$\delta x_1 = \delta x_2 = \delta x_3 = \delta x_4 = x - a_{z_1} \, \beta' \tag{4.2-4c}$$

The restoring forces exerted by the i-th isolator on the rigid body are $-K_X G_\omega^* \delta x_i$ and $-K_Z G_\omega^* \delta z_i$ along X and Z axes respectively. Equations of motion can be written by examining the free body diagram of the rigid body and isolators.

$$M \ddot{z} = -K_Z G_\omega^* \Sigma \delta z_i$$

$$I_Y \ddot{\beta}' = - K_X G_\omega^* \Sigma \delta x_i a_{z_i} + K_Z G_\omega^* \delta z_i a_{x_i}$$

$$M \ddot{x} = -K_X G_\omega^* \Sigma \delta x_i$$

where M is the mass, I_Y is the principal moment of inertia of the rigid body about Y-axis and the summation is carried over for i = 1 to 4. Using equations (4.2-1) and (4.2-4) the above equations are finally obtained as,

$$M \ddot{z} + 2K_Z G_\omega^* (2z - z_{B_1} - z_{B_3}) = F_Z e^{j\omega t} \tag{4.2-5a}$$

$$I_Y \ddot{\beta}' + (4K_X G_\omega^* a_Z^2 + 4 K_Z G_\omega^* a_X^2) \beta' - 4K_X G_\omega^* a_Z x$$

$$= K_Z G_\omega^* [2z_{B_1} - 2z_{B_3}] a_x - F_z dX e^{j\omega t} \tag{4.2-5b}$$

$$M \ddot{x} + 4K_X G_\omega^* (x - a_z \beta') = 0 \tag{4.2-5c}$$

for the isolators at 1 and 2,

$$\frac{\dot{z}_{B_1}}{\beta} + K_Z\, G_\omega^* \,(z_{B_1} - z + a_x \beta') = 0 \qquad (4.2\text{-}5d)$$

for the isolators at 3 and 4,

$$\frac{\dot{z}_{B_3}}{\beta} + K_Z\, G_\omega^* \,(z_{B_3} - z - a_x \beta') = 0 \qquad (4.2\text{-}5e)$$

where β is foundation mobility.

Assuming steady state solutions $\tilde{z} = z_o\, e^{j\omega t}$, $\tilde{x} = x_o\, e^{j\omega t}$, $\tilde{\beta}' = \beta'_o\, e^{j\omega t}$, $\tilde{z}_{B_1} = z_{B_{10}}\, e^{j\omega t}$ and $\tilde{z}_{B_3} = z_{B_{30}}\, e^{j\omega t}$ and substituting them in above equations, one can solve for z_o, x_o β'_o, $Z_{B_{10}}$ and $Z_{B_{30}}$. Response ratio for isolators can be calculated by the equation

$$R_{1,3} = \frac{z_{B_{1,3}}}{z_{B_{1,3R}}}$$

where $z_{B_{1,3}}$ are calculated as mentioned above and $z_{B_{1,3R}}$ are displacements of the foundation when isolators are not present.

Expression for R, can be finally obtained as

$$(R)_{1,3} = |(R)_F \pm (R)_C| \qquad (4.2\text{-}6)$$

where $(R)_F$ is defined as

$$(R)_F = \frac{K_Z\, G^*_\omega\, (-M\omega^2 + 4j\omega/\beta)}{[K_Z\, G^*_\omega\, (-M\omega^2 + 4j\omega/\beta) - M\omega^2 (j\omega/\beta)\;]}$$

and $(R)_C$ is defined as

$$(R)_C = \frac{a_x\, dx\, F^*_1\, F^*_2}{F^*_3\, F^*_1\, F^*_4 - F^*_5 + F^*_6}$$

where F^*_1, F^*_2, F^*_3, F^*_4, F^*_5 and F^*_6 are defined as follows:

$$F^*_1 = (-\,M\omega^2 + 4K_X\, G^*_\omega\,)$$

$$F^*_2 = K_Z\, G^*_\omega\, (-M\omega^2 + 4j\omega/\beta)$$

$$F^*_3 = (-I_Y\omega^2 + 4\, K_X\, G^*_\omega\, a_Z^2 + 4K_Z\, G^*_\omega\, a_X^2)$$

$$F^*_4 = (j\omega/\beta + K_2\, G^*_\omega)$$

$$F^*_5 = 16\, K_X^2\, G^*_\omega\, a_x\, a_z$$

$$F^*_6 = 4K_z\, G^*_\omega\, a_x^2$$

$(R)_F$ can be expressed in the nondimensional form as

$$(R)_F = \frac{1}{(1 - (\frac{\omega}{\omega_o})^{2-b_1}\, [\frac{1}{\{4 + j(\frac{\omega}{\omega_o})^{s+1}\, (\gamma + j\delta)\,\}}] \quad \frac{1}{(1+j\delta G_\omega)})} \qquad (4.2\text{-}7)$$

The relation $G^*_\omega = a_1 \omega^{b_1} (1 + j\,\delta G\omega)$ is made of use of in the above equation where ω_o is defined as the natural frequency in z-direction, $\omega_o^2 = \frac{K_z G_o}{M}$. Expressing $(R)_C$ in nondimensional parameters dx/a_x, ρ_y/a_x, a_z/a_x, K_x/K_z etc., it can be written that

$$(R)_C = \frac{(\frac{d_x}{a_x})\ H_1^*}{H_2^*\ H_3^* - H_4^* + H_5^*} \tag{4.2-8}$$

where H_1^*, H_2^*, H_3^*, H_4^*, H_5^* are defined as follows:

$$H_1^* = [4j(\frac{\omega}{\omega_o}) - (\gamma + j\delta)(\frac{\omega}{\omega_o})^{s+2}]\frac{\omega_o}{\beta}$$

$$H_2^* = (\frac{\omega}{\omega_o})^{2-b_1} \frac{1}{(1 + j\,\delta\,G)_\omega} (\frac{\rho_y}{a_x})^2 + 4(\frac{K_X}{K_Z})(\frac{a_z}{a_x})^2 + 4$$

$$H_3^* = [j + (\frac{\omega}{\omega_o})^{s+b_1+1}(\gamma + j\delta]\frac{\omega_o}{\beta}$$

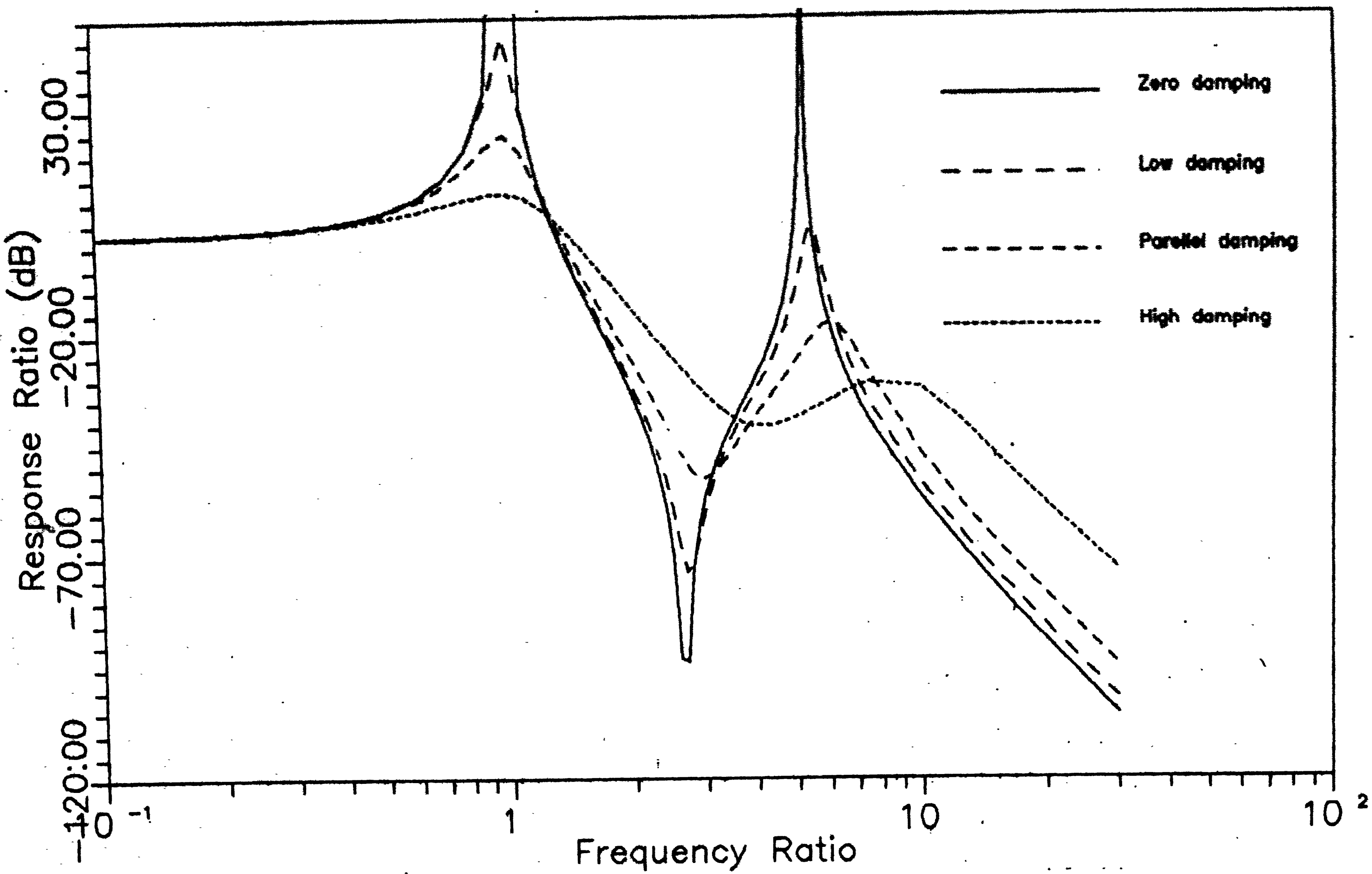
$$H_4^* = 16(\frac{K_X}{K_Z})(\frac{a_z}{a_x}) \frac{1}{[4 - (\frac{\omega}{\omega_o})^{2-b_1} \frac{1}{(1+ j\,\delta\,G)_\omega}(\frac{K_Z}{K_X})}$$

$$H_5^* = \frac{4}{\omega_o^2 M (\frac{\omega}{\omega_o})^{b_1} (\frac{K_X}{K_Z}) [-(\frac{\omega}{\omega_o})^{2-b_1} \frac{1}{(1+j\delta G)_\omega}(\frac{K_Z}{K_X}) + 1]}$$

$$\omega_o^2 = \frac{K_Z G_o}{M}$$

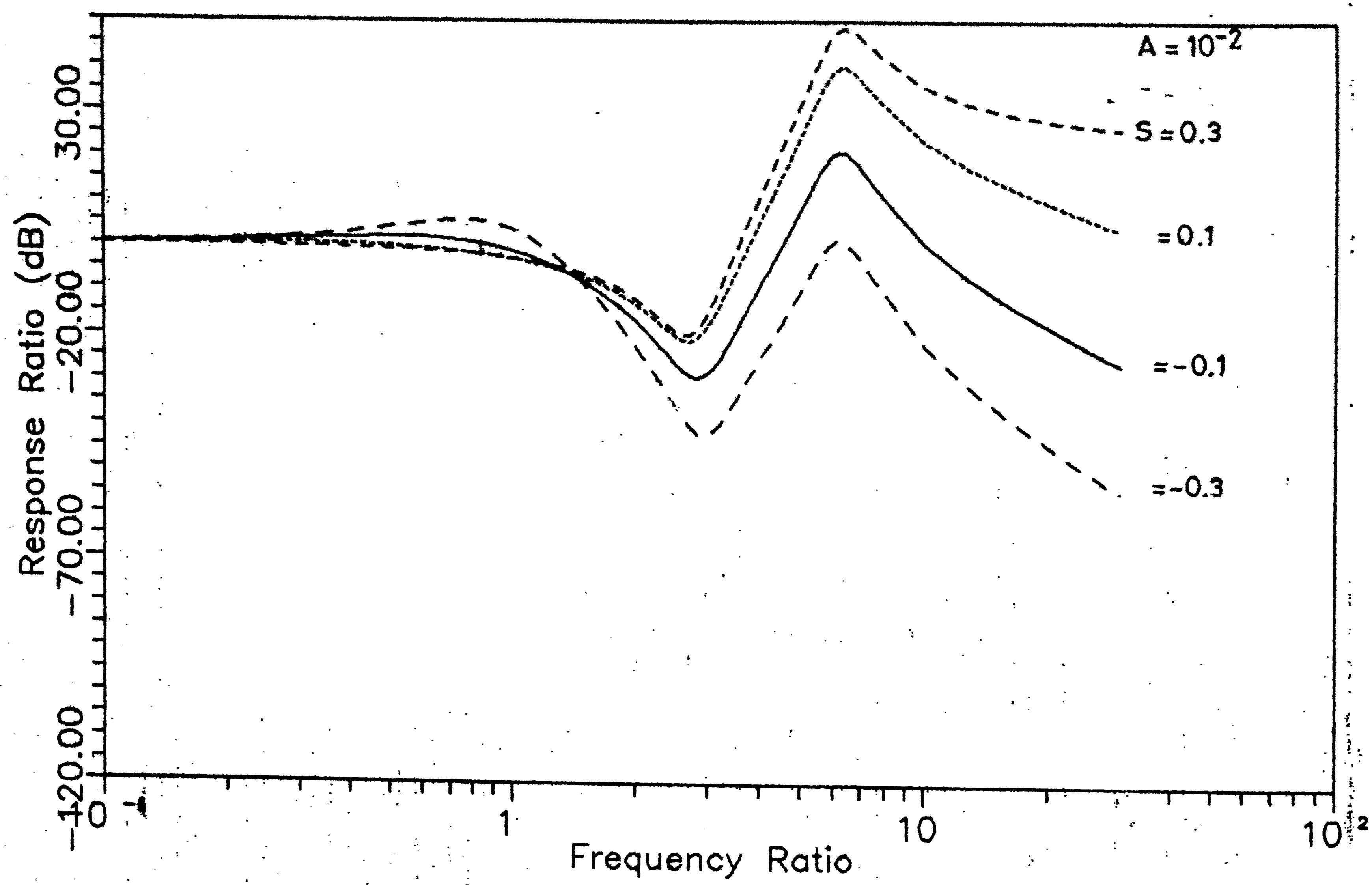
$(R)_1$ is plotted for low damping, high damping and parallel combination of the two rubbers for not a very mobile foundation in Fig. 4.2-2. As expected two peaks, one corresponding to coupled motion and one corresponding to translation in z-direction are observed. Data needed for this plot is taken from reference [18] and results of the present analysis are compared with that given in the above reference when isolator damping is ignored and isolator stiffness is independent of frequency. They are found to be in agreement with each other. The damping and frequency dependence of stiffness of isolator significantly affect the response ratio curve as shown in the figure. The position of a resonance peak and overall level of the curve are also shifted to higher values as observed from figure for a high damping rubber.

The effect of nonrigidity of the foundation on the system considered above is shown in Fig. 4.2-3 and Fig. 4.2-4 for different types of nonrigid foundation. Response ratios for the isolator at position 1, given by eqn. (4.2-9) are plotted for various values of foundation mobility. Foundation mobility data for different types of floor constructions are taken from reference [32]. The effect of increase in foundation mobility is to reduce the peak value at the first resonance and to increase the levels of transmissibility at higher frequencies. It can be seen from the figure that the shape of the curve at second resonance is not affected by



Zero damping
Low damping
Parallel damping
High damping
30.00
-20.00
-70.00
-120.00
Response Ratio (dB)
10 -1
1
10
10 2
Frequency Ratio

Fig. 4.2-2



A = 10^-2
S = 0.3
= 0.1
= -0.1
= -0.3
30.00
-20.00
-70.00
-120.00
Response Ratio (dB)
10^-1
1
10
10^2
Frequency Ratio

Fig. 4.2-4

foundation mobility, but the curve is simply shifted to higher levels. These figures show that ignoring the effects of the couple may be significant when the foundation is flexible.

Analysis of wave effects in isolators in two point excitation case is carried out, but the results are not presented here as no new conclusions can be drawn.

CHAPTER V

CONCLUSIONS

5.1 Conclusions

The following conclusions can be drawn from the present work:

(1) Frequency dependent properties of rubber like materials play an important role in the analysis of vibration isolation systems.

(2) Parallel mounting is superior to either low or high damping mountings in all cases.

(3) The performance of the vibration isolation system are normally overestimated if the foundation mobility is not considered.

(4) Unidirectional multi degrees-of-freedom vibration isolation systems offer advantages at high frequencies over simple systems. However, with increasing number of elements more resonance peaks are introduced and the performance is offset by the increasing peak values of wave resonances for low damping isolators.

(5) The effect of placing steel spacers beyond a certain number in rubber mounts may prove to be counter effective in reducing the wave effects in isolators. Therefore, care should be exercised in selecting the number of steel spacers.

(6) The effect of the exciting couple on a rigid body may be predominant when the foundation has finite-impedance.

(7) A mobility technique when used in conjunction with periodicity offers elagant solutions as compared to the four pole parameter technique for analysing unidirectional periodic vibration isolation systems specially for a large number periodic elements.

5.2 Suggestions for Further Work

1. Modelling of the foundation should be improvised to include the resonances of foundations.

2. Complicated geometries of rubber isolators should be analysed for studying the wave effects in isolators.

3. Experimental work should be carried out to validate the advantages of parallel mounting and periodic isolators.

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APPENDIX A

SOME DEFINITIONS

A.1

Response Parameter R	Standard R/F	Inverse F/R
Displacement	Receptance Admittance Dynamic flexibi- lity Dynamic Compliance	Dynamic Stiffness
Velocity	Mobility	Mechanical impedance
Acceleration	Inertance accelerance	Apparent mass

(Produced from Reference [47]).

A.2 Rubber isolator is also referred to as antivibration mount, compliant isolator, rubber mount, rubber pad, rubber element, rubber spring, elastomeric spring.

A.3 Nonrigid foundation is also referred to as flexible seating structure, foundation substructure.

APPENDIX-B

EVALUATION OF MOBILITIES

The solution of governing equation of longitudinal vibrations of a long rod shown in Figs. B.1 and B.2 can be expressed as [1],

$$\xi^* = C^* \operatorname{Sin} n^* x + D^* \operatorname{Cos} n^* x \qquad \text{B.1}$$

where C^* and D^* depend on boundary conditions of the long rod.

$$\text{and } n^* = \omega(\rho/E^*_\omega)^{1/2} \qquad \text{B.2}$$

From Eq. (B.1) it can be written that

$$\frac{\partial \xi^*}{\partial x} = n^* [C^* \operatorname{Cos} n^* x - D^* \operatorname{Sin} n^* x] \qquad \text{B.3}$$

Case (i)

From Fig. B.1, if $F_o = 1$, then the mobilities are defined as

$$\beta_{\ell\ell} = \dot{\xi}^*\Big|_{x=0} = j\omega\xi^*\Big|_{x=0} \qquad \text{B.4a}$$

and

$$\beta_{r\ell} = \dot{\xi}^*\Big|_{x=\ell} = j\omega\xi^*\Big|_{x=\ell} \qquad \text{B.4b}$$

where $\dot{\xi}^* = \dfrac{\partial \xi^*}{\partial t}$

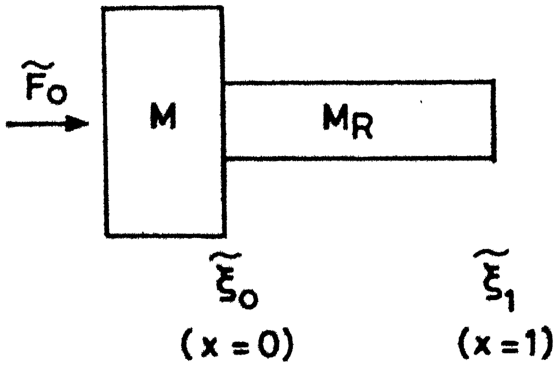

Fig. B.1

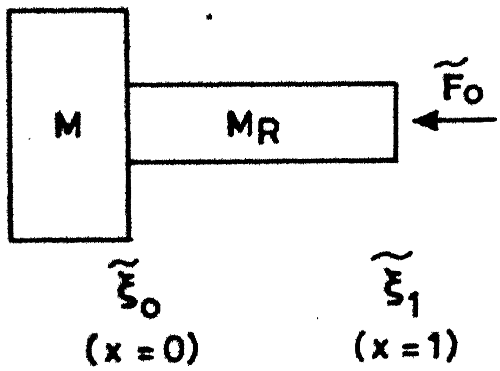

Fig. B.2

Evaluation of Mobilities.

second subscript to that of the force.

The boundary conditions are

1. At $x = \ell$ $\quad \dfrac{\partial \xi^*}{\partial x} = 0$ B.5a

and

2. $-M\omega^2 \xi^*\big|_{x=0} = \tilde{F}_o - \tilde{F}_1$ B.5b

where $\tilde{F}_1 = - aE^*_\omega \dfrac{\partial \xi^*}{\partial x}\Big|_{x=0}$

Solving for D^* and C^* from eqns. B.5, B.1 and B.2 and finding $\beta_{\ell\ell}$ and $\beta_{r\ell}$ from eqns. B.4, one can write that

$$\beta_{\ell\ell} = \frac{-j\omega \operatorname{Cos} n^*\ell}{a E^*_\omega n^* \theta^*} \qquad \text{B.6a}$$

and

$$\beta_{r\ell} = \frac{-j\omega}{a E^*_\omega n^* \theta^*} \qquad \text{B.6b}$$

where $\theta^* = \gamma_2 (n^*\ell) \operatorname{Cos}(n^*\ell) + \operatorname{Sin}(n^*\ell)$ B.7

$\gamma_2 = M/M_R$ where M is the end mass and M_R is the mass of the rod $= a\rho\ell$

$E^*_\omega = E_\omega(1 + j\delta E_\omega)$

a is the cross-sectional area of the rod, ρ, is density and ℓ is length.

Case (ii)

For the Fig. B.2 the boundary conditions can be similarly obtained as

1. At $x = \ell$, $F = -a E_\omega^* \left.\frac{\partial \xi^*}{\partial x}\right|_{x=\ell}$ B.8a

2. At x = 0, from the free body diagram of the mass and rod

$$-M\omega^2 \xi^*\big|_{x=0} = a E_\omega^* \left.\frac{\partial \xi^*}{\partial x}\right|_{x=0} \qquad \text{B.8b}$$

If $F_o = 1$,

The mobilities β_{rr}, $\beta_{\ell r}$ are defined as follows

$$\beta_{rr} = \dot{\xi}^*\big|_{x=\ell} = j\omega\xi^*\big|_{x=\ell} \qquad \text{B.9a}$$

$$\beta_{\ell r} = \dot{\xi}\big|_{x=0} = j\omega\xi^*\big|_{x=0} \qquad \text{B.9b}$$

Solving for D^*, C^* from eqns. (B.8), (B.1) and (B.2) and finding β_{rr}, $\beta_{\ell r}$ from eqn. (B.9) one can write that

$$\beta_{rr} = \frac{j\ \omega\eta^*}{a\ E_\omega^*\ n^*\ \theta^*} \qquad \text{B.10a}$$

$$\beta_{\ell r} = \frac{j\omega}{a\ E_\omega^*\ n^*\ \theta^*} \qquad \text{B.10b}$$

where $\eta^* = -\gamma_2\ (n^*\ell)\ \mathrm{Sin}(n^*\ell) + \mathrm{Cos}(n^*\ell)$ B.11

denoting $F^* = j\left(\frac{\omega}{\omega_o}\right)^{1-b_1} \frac{1}{(1+j\delta E_\omega)} \frac{1}{\gamma_2 M_R \omega_o} \frac{1}{(n^*\ell)\theta^*}$

β's can be written in the form

$$\beta_{rr} = F^* \eta^*$$

$$\beta_{r\ell} = -F^*$$

$$\beta_{\ell r} = F^*$$

$$\beta_{\ell\ell} = -F^* \operatorname{Cos}(\eta^* \ell)$$

where $(\eta^*\ell)$ is defined in section 3.4.

E^*_ω is considered as $a_1 \omega^{b_1} (1 + j\delta E_\omega)$. It should be pointed out that while evaluating the mobilities in section 3.5-2, the mass ratio in expressions θ^* and η^* of eqns. (B.7) and (B.11) should be considered as M_S/M_R. This is because in section 3.5-2, the end mass of the periodic element is M_S. This ratio M_S/M_R is referred to as γ_1.